Final Exam
Course SCE1106 Control theory with implementation (theory part)
Thursday January 9th 2007
kl. 9.00-12.00

November 19, 2008

Task 1 (14%): Frequency analysis

Given a feedback system as illustrated in Figure 1.

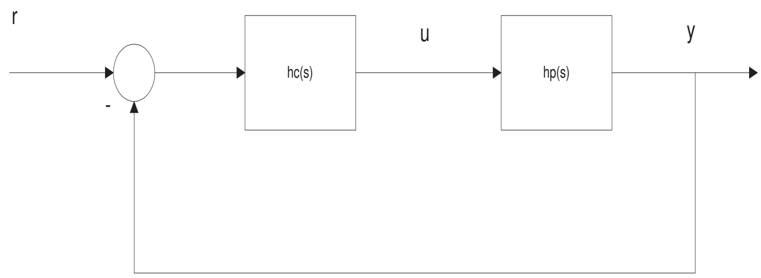

Figure 1: Standard feedback control system.

a) Write down an expression for the loop transfer function, $h_0(s)$.

b) Consider an PID controller on cascade form, $h_c(s)$, and a process, $h_p(s)$, given by

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s}(1 + T_d s), \quad h_p(s) = k \frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)}, \tag{1}$$

where $T_1 > T_2 > 0$. Chose the integral time T_i, and the derivative time, T_d, such that the loop transfer function can be written as follows

$$h_0(s) = k_0 \frac{e^{-\tau s}}{s}. \tag{2}$$

Write also down the expression for k_0.

c)

- Write the frequency response of the loop transfer function in (2) on polar form, i.e., such that

$$h_0(j\omega) = |h_0(j\omega)| e^{j\angle h_0(j\omega)}. \tag{3}$$

- What is the size $|h_0(j\omega)|$ denoted?
- What is the size $\angle h_0(j\omega)$ denoted?

d) Find the phase crossover frequency, ω_{180}, for the system.

e) Find a proportional gain, K_p, such that the closed loop system have a Gain Margin, $GM = \frac{3}{2}$.

f)

- What is the definition of, and how can the gain crossover frequency, ω_c, be computed?
- What is the gain crossover frequency, ω_c, for the above feedback control system?

g)

- What is the definition of, and how can the phase margin, PM, be computed?
- What is the phase margin, PM, for the above feedback control system?

Task 2 (6%): Ziegler Nichols method

Consider a process, $h_p(s)$, given by

$$h_p(s) = k\frac{1-\tau s}{(1+T_1 s)(1+T_2 s)}. \tag{4}$$

where $T_1 > T_2 > 0$, and a PI controller, $h_c(s)$, given by

$$h_c(s) = K_p\frac{1+T_i s}{T_i s}. \tag{5}$$

We want to find the PI controller parameters K_p and T_i by using the Ziegler Nichols method.

a)

- Show how the critical gain K_{cu} for the above system can be computed, for use in the Ziegler Nichols method.
- Commenting upon the relationship between the Gain crossover frequency, ω_c, and the Phase crossover frequency, ω_{180}, when the closed loop system have the critical gain, K_{cu}.

b) Define the PI controller parameters in the Ziegler Nichols method as a function of K_{cu} and ω_{180}.

Task 3 (10%): PID regulator

Given a PID controller in the Laplace plane

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s} + K_p T_d s, \tag{6}$$

such that the control is generated by

$$u(s) = h_c(s) e(s) \tag{7}$$

where $e(s) = r - y(s)$ is the control error. We are assuming a constant reference signal, r, in this task.

a) Write down a continuous state space model for the PID controller in Equations (6) and (7).

b) Find a discrete time state space model for the PID controller in Step 2a) above. Use the explicit Euler method for discretization.

c) Write the discrete time PID controller in Step 2b) above on so called incremental (deviation) form, i.e. in such a way that the control is generated by the formula

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2 (y_k - 2y_{k-1} + y_{k-2}). \tag{8}$$

You should also write down the expressions for the parameters g_0, g_1 and g_2.

d) Use the Trapezoid method for discretization in Step 2b) and find the corresponding PID controller on incremental form as in Step 2c) above.

Task 4 (5%): Various questions

a) What is a Smith predictor? When is it suitable to use a Smith predictor? Sketch a block diagram of a system controlled by a Smith predictor.

b) What is meant by RGA analysis? Write down a formula for computing the RGA matrix. What can the RGA matrix be used for?

c) What is a non-minimum phase system? Five an example of a non-minimum phase system.

d) Give a definition of a non-minimum phase system.

e) What is the solution, $x(t)$, for the linear continuous time state equation

$$\dot{x} = Ax + Bu \tag{9}$$

where the initial state, $x(t_0)$ is known ?