# Final Exam
# Course SCE1106 Control theory with implementation (theory part)
# Thursday January 9th 2007
# kl. 9.00-12.00

December 11, 2008

# Task 1 (13%): Frequency analysis

Given a feedback system as illustrated in Figure 1.

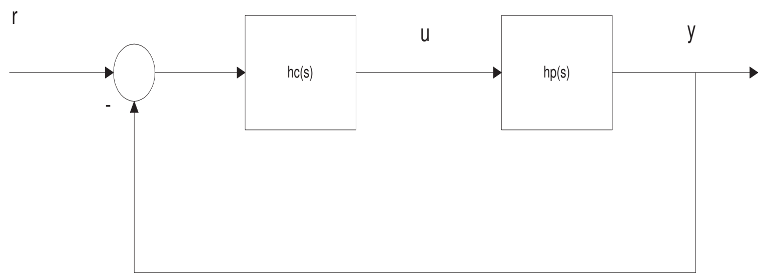


Figure 1: Standard feedback control system.

**a)** Write down an expression for the loop transfer function, $h_0(s)$.

**b)** Consider an PID controller on cascade form, $h_c(s)$, and a process, $h_p(s)$, given by

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s}(1 + T_d s), \quad h_p(s) = k \frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)}, \tag{1}$$

where $T_1 > T_2 > 0$. Chose the integral time $T_i$, and the derivative time, $T_d$, such that the loop transfer function can be written as follows

$$h_0(s) = k_0 \frac{e^{-\tau s}}{s}. \tag{2}$$

Write also down the expression for $k_0$.

**c)**

- Write the frequency response of the loop transfer function in (2) on polar form, i.e., such that

$$h_0(j\omega) = |h_0(j\omega)| e^{j\angle h_0(j\omega)}. \tag{3}$$

- What is the size $|h_0(j\omega)|$ denoted?
- What is the size $\angle h_0(j\omega)$ denoted?

**d)** Find the phase crossover frequency, $\omega_{180}$, for the system.

**e)** Find a proportional gain, $K_p$, such that the closed loop system have a Gain Margin, $GM = \frac{3}{2}$.

**f)**

- What is the definition of, and how can the gain crossover frequency, $\omega_c$, be computed?
- What is the gain crossover frequency, $\omega_c$, for the above feedback control system?

# Task 2 (6%): Ziegler Nichols method

Consider a process, $h_p(s)$, given by

$$h_p(s) = k\frac{1-\tau s}{s(1+T_1 s)}. \tag{4}$$

where $T_1 > 0$, and a PI controller, $h_c(s)$, given by

$$h_c(s) = K_p\frac{1+T_i s}{T_i s}. \tag{5}$$

We want to find the PI controller parameters $K_p$ and $T_i$ by using the Ziegler Nichols method.

**a)**

- Show how the critical gain $K_{cu}$ for the above system can be computed, for use in the Ziegler Nichols method.
- Commenting upon the relationship between the Gain crossover frequency, $\omega_c$, and the Phase crossover frequency, $\omega_{180}$, when the closed loop system have the critical gain, $K_{cu}$.

**b)** Define the PI controller parameters in the Ziegler Nichols method as a function of $K_{cu}$ and $\omega_{180}$.

# Task 3 (8%): PID regulator

Given a PID controller in the Laplace plane

$$h_c(s) = K_p\frac{1+T_i s}{T_i s} + K_p T_d s, \tag{6}$$

such that the control is generated by

$$u(s) = h_c(s)e(s) \tag{7}$$

where $e(s) = r - y(s)$ is the control error. We are assuming a constant reference signal, $r$, in this task.

**a)** Write down a continuous state space model for the PID controller in Equations (6) and (7).

**b)** Find a discrete time state space model for the PID controller in Step 2a) above. Use the explicit Euler method for discretization.

**c)** Write the discrete time PID controller in Step 2b) above on so called incremental (deviation) form, i.e. in such a way that the control is generated by the formula

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2(y_k - 2y_{k-1} + y_{k-2}). \tag{8}$$

You should also write down the expressions for the parameters $g_0$, $g_1$ and $g_2$.

## Task 4 (8%): Feed-forward controller and Smith predictor

Given a system described by the following transfer function model

$$y = h_u(s)u + h_v(s)v \tag{9}$$

**a)** Assume that the system is controlled by a controller

$$u = h_c(s)e + u_f \tag{10}$$

where $h_c$ is the controller transfer function, $e = r - y$ is the controller input and $u_f$ is a feed-forward signal.

- Find the ideal feed-forward controller, i.e., find the feed-forward signal, $u_f$, such that the disturbance $v$ does not influence upon the output $y$.

**b)** Answer the following:

- When may it make sense to use a Smith predictor?
- Sketch a block diagram of a system controlled by a Smith predictor.