Final Exam
Course SCE1106 Control theory with implementation (theory part)
Monday December 7th 2009
kl. 9.00-12.00

December 9, 2010

Task 1 (12%):
PID-control, the SIMC method

Consider a process described by the transfer function model

$$y = h_p(s)u. \tag{1}$$

The process are to be controlled by a controller of the form

$$u = h_c(s)(r - y). \tag{2}$$

The feedback control system is illustrated in Figure (1).

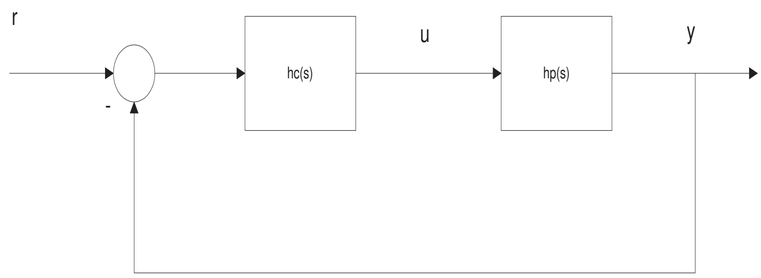

Figure 1: Standard feedback control system.

a) Consider the feedback control system in Figure (1).

- Find the transfer function from the reference, r, to the output measurement, y, i.e., find the transfer function

$$\frac{y}{r} = h_{ry}(s) \tag{3}$$

  where $h_{ry}(s)$ is the transfer function from r to y.

- Find an expression for the transfer function, $h_c(s)$, for the controller as a function of the ratio $\frac{y}{r}$ and the transfer function for the process, $h_p(s)$.

We will in the following subtasks specify that the set point response from the reference, r, to the output, y, should be given by

$$\frac{y}{r} = \frac{1 - \tau s}{1 + T_c s} \tag{4}$$

where T_c is a user specified time constant and $\tau \geq 0$ is the time delay.

b) Assume that the process, $h_p(s)$, is modelled by a 2nd order transfer function given by

$$h_p(s) = k\frac{1 - \tau s}{(1 + T_1 s)(1 + T_2 s)}, \tag{5}$$

where $T_1 > T_2 > 0$.

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

c) Assume that the process, $h_p(s)$, is modelled by a 2nd order oscillating process of the form

$$h_p(s) = k\frac{1 - \tau s}{\tau_0^2 s^2 + 2\tau_0 \xi s + 1}. \tag{6}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

d) Assume that the process is modeled by a pure time delay, i.e. with a process model

$$h_p(s) = ke^{-\tau s}. \tag{7}$$

Find the controller $h_c(s)$ by the Skogestad method. What type of controller is this?

e) Given a process with a model as follows

$$y = h_p(s)u, \tag{8}$$

where

$$h_p(s) = k\frac{e^{-\tau s}}{(1 + Ts)^8}. \tag{9}$$

and $T > 0$.

Use the half rule for model reduction and formulate a 1st order model approximation of the form

$$h_p(s) = k\frac{e^{-\tau s}}{1 + Ts}, \tag{10}$$

for the transfer function in Equation (9).

f) Assume that the process, $h_p(s)$, is modeled by a 1st order model given by

$$h_p(s) = k\frac{e^{-\tau s}}{1 + T_1 s}. \tag{11}$$

What is the SIMC PI controller setting for this process model. Specify the SIMC setting for the PI controller parameters K_p and T_i in the controller transfer function $h_c(s)$.

Task 2 (25%): Frequency analysis

Given a feedback system as illustrated in Figure 2.

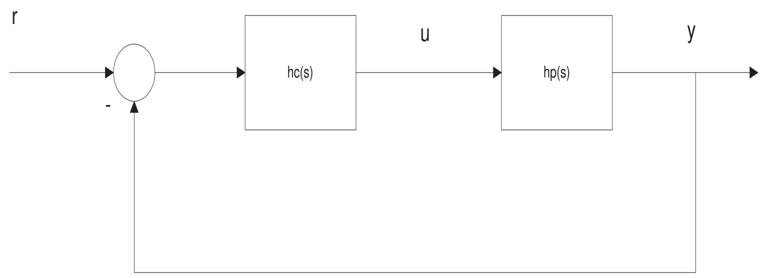

Figure 2: Standard feedback control system.

a) Consider an PI controller, $h_c(s)$, and an integrating plus time delay process, $h_p(s)$, given by

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s}, \quad h_p(s) = k \frac{e^{-\tau s}}{s}, \tag{12}$$

where K_p and T_i is the PI controller parameters, k is the gain velocity (slope of the integrator) and τ the time delay.

- Write down an expression for the loop transfer function, $h_0(s)$.

b) Assume in this subtask 2b) that we use an approximation $e^{-\tau s} \approx 1$ for the time delay (same as neglecting the time delay in the model, Eq. (12).

The set-point response transfer function may then be written as

$$\frac{y}{r} = \frac{h_0}{\pi(s)} \tag{13}$$

where the characteristic polynomial $\pi(s)$ may be written on standard 2nd order form as follows

$$\pi(s) = \tau_0^2 s^2 + 2\zeta\tau_0 s + 1. \tag{14}$$

Here τ_0 is the response time and ζ the relative damping coefficient.

- Find expressions for the coefficients τ_0^2 and $2\zeta\tau_0$ in the characteristic polynomial Eq. (14) as a function of the PI controller parameters K_p, T_i and the gain velocity parameter k.

- Assume that we prescribe a unit relative damping, i.e. $\zeta = 1$. Find expressions for the PI controller parameters K_p and T_i as a function of a prescribed response time $\tau_0 > 0$.

c)

- Find an expression for the frequency response of the loop transfer function $h_0(s)$ found in subtask 2a), on polar form, i.e., such that

  $$h_0(j\omega) = |h_0(j\omega)|e^{j\angle h_0(j\omega)}. \tag{15}$$

  Find expressions for the magnitude $|h_0(j\omega)|$ and the phase angle $\angle h_0(j\omega)$.

  Note: You should in this subtask 2c) use the integrator plus time delay model as in Eq. (12).
- Describe the Bode stability criterion !

d)

- Find an expression for the phase crossover frequency, ω_{180}, for the closed loop system ?
- Find an expression for the Gain Margin (GM) ?
- What is the interpretation of the GM ?

e)

- What is the definition of, and how can the gain crossover frequency, ω_c, be computed?
- Find an expression for the Phase Margin (PM) ?
- What is the interpretation of the PM ?

Task 3 (10%): PID controller

Given a PID controller in the Laplace plane

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s} + K_p T_d s = K_p + \frac{K_p}{T_i s} + K_p T_d s, \tag{16}$$

such that the control is generated by

$$u(s) = h_c(s)e(s) \tag{17}$$

where $e(s) = r - y(s)$ is the control error. We are assuming a constant reference signal, r, in this task.

a) Write down a continuous state space model for the PID controller in Equations (16) and (17).

b) Given a linear system

$$\dot{x} = Ax + Bu \tag{18}$$

where $x(t_0)$ is a specified initial state vector.

Write down an expression for the solution, $x(t)$?

c) Use the result from task 3b) above in order to find an expression for the control signal, $u(t)$. Tips: Here you should find a formulation of, $u(t)$, in terms of among others an integral expression.

d) Find a discrete time state space model for the PID controller in Step 3a) above. Use the explicit Euler method for discretization.

e) Write the discrete time PID controller in Step 3d) above on so called incremental (deviation) form, i.e. in such a way that the control is generated by the formula

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2(y_k - 2y_{k-1} + y_{k-2}). \tag{19}$$

You should also write down the expressions for the parameters g_0, g_1 and g_2.

Task 4 (3%): Smith predictor

Given a system described by the following transfer function model

$$y = h_u(s)u + h_v(s)v \tag{20}$$

a) Answer the following:

- When may it make sense to use a Smith predictor?
- Sketch a block diagram of a system controlled by a Smith predictor.

b) Find an expression for the transfer function for the closed loop set-point response, $\frac{y}{r} = h_{ry}$, i.e. the transfer function from the reference, r, to the output, y. You may put the disturbance equal to zero, i.e., $v = 0$ in the derivation.