

Final Exam
Course SCE1106 Control theory with
implementation (theory part)
Wednesday December 18, 2014
kl. 9.00-12.00
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December 16, 2014

Task 1 (24%): PID-control, the SIMC method

Consider a process described by the transfer function model

$$y = h_p(s)u. \quad (1)$$

The process is to be controlled by a controller of the form

$$u = h_c(s)(r - y). \quad (2)$$

The feedback control system is illustrated in Figure (1).

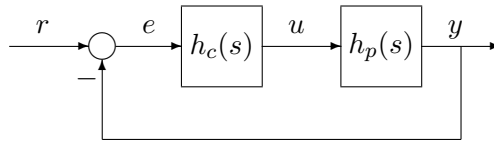


Figure 1: Standard feedback system. Plant described by a transfer function model $h_p(s)$ and controller with transfer function $h_c(s)$.

a) Consider the feedback control system in Figure (1).

- Find the transfer function $h_{ry}(s)$ from the reference, r , to the output measurement, y , in the transfer function model

$$y = h_{ry}(s)r. \quad (3)$$

I.e. find an expression for $h_{ry}(s)$?

We will in the following subtasks specify that the set point response from the reference, r , to the output, y , should be given by

$$y = \frac{e^{-\tau s}}{1 + T_c s} r \quad (4)$$

where $T_c \geq \tau$ is a user specified time constant and $\tau > 0$ is the time delay.

b)

- Find an expression for the controller transfer function, $h_c(s)$, as a function of the time delay transfer function $e^{-\tau s}$ and the model transfer function $h_p(s)$ for the process.
- Use the simple approximation $e^{-\tau s} \approx 1 - \tau s$ and find the resulting controller transfer function $h_c(s)$?

c) Given a process with a 2nd order model as follows

$$y = h_p(s)u, \quad (5)$$

where

$$h_p(s) = 20 \frac{1}{(1 + 10s)(1 + s)}. \quad (6)$$

- Use the half rule for model reduction and formulate a 1st order model approximation of the form

$$h_p(s) = k \frac{e^{-\tau s}}{1 + T_1 s}, \quad (7)$$

for the transfer function in Equation (6). Find the gain k , the time delay τ and the time constant T_1 in the model approximation Equation (7) ?

- Find the controller $h_c(s)$ by the SIMC (Skogestad) method. What type of controller is this?

d) Assume that the process, $h_p(s)$, is modeled by a 2nd order process of the form

$$h_p(s) = k \frac{e^{-\tau s}}{\tau_0^2 s^2 + 2\tau_0 \xi s + 1}. \quad (8)$$

- What are the definitions for the parameters ξ and τ_0 in in the model (8).
 - When is the process oscillating ?
 - Find the controller $h_c(s)$ by the SIMC (Skogestad) method.
 - What type of controller is this?
- e) Specify the poles in the system described by Equation (8) for the following two cases:
- $\xi = 1$.
 - $\xi > 1$.
- f) Consider a system where the speed of response $\tau_0 = 0$ ("infinite" fast system) in Equation (8) such that the system can be described by a pure steady state plus time delay process

$$h_p(s) = k e^{-\tau s}. \quad (9)$$

- Find a controller by Skogestad method.
- What type of controller is this ?

Task 2 (6%): Frequency analysis

Given a feedback system as illustrated in Figure 2.

- a) Consider an PI controller, $h_c(s)$, and an integrating plus time delay process, $h_p(s)$, given by

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s}, \quad h_p(s) = k \frac{e^{-\tau s}}{s}, \quad (10)$$

where K_p and T_i are the PI controller parameters, k is the gain velocity (slope of the integrator) and τ the time delay.

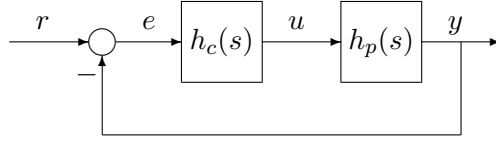


Figure 2: Standard feedback system. Plant described by a transfer function model $h_p(s)$ and controller transfer function $h_c(s)$.

- Write down an expression for the loop transfer function, $h_0(s)$.
- b) Assume in this subtask 2b) that we use an approximation $e^{-\tau s} \approx 1$ for the time delay (same as neglecting the time delay in the model, Eq. (10)).

The set-point response transfer function may then be written as

$$\frac{y}{r} = \frac{h_0}{\pi(s)} \quad (11)$$

where the characteristic polynomial $\pi(s)$ may be written on standard 2nd order form as follows

$$\pi(s) = \tau_0^2 s^2 + 2\zeta\tau_0 s + 1. \quad (12)$$

Here τ_0 is the response time and ζ the relative damping coefficient.

- Find expressions for the coefficients τ_0^2 and $2\zeta\tau_0$ in the characteristic polynomial Eq. (12) as a function of the PI controller parameters K_p , T_i and the gain velocity parameter k .
- Assume that we prescribe a unit relative damping, i.e. $\zeta = 1$. Find expressions for the PI controller parameters K_p and T_i as a function of a prescribed response time $\tau_0 > 0$.

Task 3 (12%): Frequency analysis

Given a system which can be described by a first order time delay model

$$h_p(s) = K \frac{e^{-\tau s}}{1 + Ts} \quad (13)$$

where $T > 0$ is the time constant and $\tau > 0$ the time delay.

- a) Answer the following: When can we approximate the model Eq. (13) as an integrating plus time delay model ?

$$h_p(s) = k \frac{e^{-\tau s}}{s} \quad (14)$$

Find an expression for the velocity gain (slope of integrator) k ?

b)

Find an expression for the frequency response of the loop transfer function $h_0(s)$ found in subtask 3a), on polar form, i.e., such that

$$h_0(j\omega) = |h_0(j\omega)|e^{j\angle h_0(j\omega)}, \quad (15)$$

assuming a P-controller is used.

Find expressions for the magnitude $|h_0(j\omega)|$ and the phase angle $\angle h_0(j\omega)$.

Note: You should in this subtask 3b) use the integrator plus time delay model as in Eq. (14).

c) Answer the following: Use a P controller and the integrating plus time delay model as in Eq. (14)

and find the ultimate gain K_{cu} and the ultimate period $P_u = \frac{2\pi}{\omega_{180}}$.

d) Answer the following: Show how you can estimate the model parameters k and τ in Eq. (14) from the ultimate gain K_{cu} and the ultimate period P_u . ?

Task 4 (9%): PID controller

Given a PID controller in the Laplace plane

$$h_c(s) = K_p \frac{1 + T_i s}{T_i s} + K_p T_d s = K_p + \frac{K_p}{T_i s} + K_p T_d s, \quad (16)$$

such that the control is generated by

$$u(s) = h_c(s)e(s) \quad (17)$$

where $e(s) = r - y(s)$ is the control error. We are assuming a constant reference signal, r , in this task.

a) Write down a continuous state space model for the PID controller in Equations (16) and (17).

b) Find a discrete time state space model for the PID controller in Step 4a) above.

Use the explicit Euler method for discretization.

c)

In this subtask you should use the explicit Euler method for discretization.

Find a discrete time PID controller in Step 4a) above on so called velocity (incremental, deviation) form, i.e. in such a way that the control is generated by the formula

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2 (y_k - 2y_{k-1} + y_{k-2}). \quad (18)$$

You should also write down the expressions for the parameters g_0 , g_1 and g_2 .

Task 5 (9%): Ratio Control

Given a system with two flow variables q_A and q_B as illustrated in Figure 3. We want to control the flow variables such that the ratio

$$k = \frac{q_A}{q_B} \quad (19)$$

is constant.

Propose a ratio control system, and sketch a block diagram of the ratio control system ?

The following should be assumed:

- Use two PID controllers h_c^A and h_c^B , one controller for each flow variables q_A and q_B , respectively.
- Assume that the flow variables may be manipulated by valves, one control input u_A to a valve A for flow variable q_A and one valve B such that u_B is the control variable to manipulate flow variable q_B .
- The flow variables q_A and q_B are measured at the input side of the valves.
- Define reference variables r_A for flow q_A and r_B for flow q_B . Specify the relationship $r_A = kr_B$ and that the set-point (reference variable) r_B is the resulting control variable in the entire ratio control system.

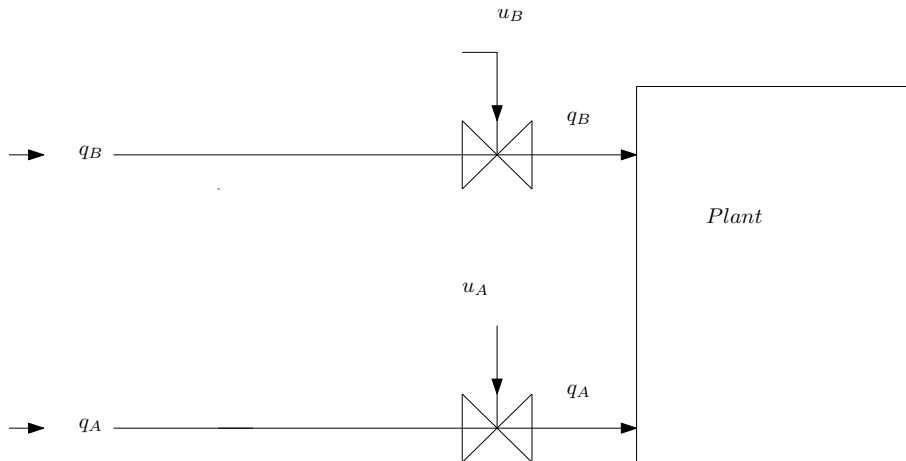


Figure 3: System