Master study
Systems and Control Engineering
Department of Technology
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SCE1106 Control Theory

Exercise 10

Task 1: Discretization of process model

Given a process or system described by the transfer function model

$$y = h_p(s)u \tag{1}$$

where

$$h_p(s) = \frac{k}{(1+T_1 s)(1+T_2 s)} \tag{2}$$

and where $k = 0.5$, $T_1 = 1$ and $T_2 = 0.5$.

a) Find a continuous state space model from the transfer function model above.

b) Use the explicit Euler method for discretization and find a discrete state space model for the model in step 1a) above.

c) Use the trapezoid method for discretization and find a discrete state space model for the model in step 1a) above.

d) Mention some advantages and perhaps disadvantages of the Trapezoid method compared with the explicit Euler method.

Tips: The reader will solve this task more easily by reading Example 5.2 in the lecture notes.

Task 2: Discretization of PID controller

Given a PID-controller

$$u = h_c(s)e \tag{3}$$

where $e = r - y$ and

$$h_c(s) = K_p \frac{1+T_i s}{T_i s} + K_p T_d s. \tag{4}$$

a) Find a continuous state space model for the PID controller. What is a reasonable initial value for the controller state, $z(t_0)$?

b)

- Use the explicit Euler method and find a discrete time state space model for the PID controller, from the continuous state space model found in step 2a) above.
- From the discrete PID controller you should find a PID controller on deviation form, i.e. a PID controller formulation on the following form

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2(y_k - 2y_{k-1} + y_{k-2}) \tag{5}$$

  Find expressions for the parameters g_0, g_1 and g_2.

c)

- Use the trapezoid method and find a discrete time state space model for the PID controller, from the continuous state space model found in step 2a) above.
- From the discrete PID controller you should find a PID controller on deviation form, i.e. a PID controller formulation on the following form

$$u_k = u_{k-1} + g_0 e_k + g_1 e_{k-1} + g_2(y_k - 2y_{k-1} + y_{k-2}) \tag{6}$$

  You should obtain the same answer as in Equation 11.5 in the book by Balchen.

Note: The parameters g_0, g_1 og g_2 will naturally be different in step b) and c) above.

Task 3: Simulation

Assume that there is a transport delay in addition to the process model in task 1 such that the process can be described by

$$y = h_p(s)u \tag{7}$$

where

$$h_p(s) = \frac{k}{(1+T_1 s)(1+T_2 s)} e^{-\tau s} \tag{8}$$

and where $k = 0.5$, $T_1 = 1$, $T_2 = 0.5$ and $\tau = 2$. Furthermore, assume that the system is to be controlled by a discrete PID controller as found in Task 2.
Write a MATLAB script program for simulation of the closed loop control system.
Use the discrete time model for the process as found in Task 1 and a discrete time formulation of the PID controller on deviation form as found in Task 2. A discrete PID controller on state space form is used in earlier exercises.
The only difference in the process model in this task and the model in Task 1, is the additional transport delay. Hence, we can use the model found in Task 1 by adding a transport delay.

A time delay can easily be implemented by using an array. For each time instant the variable which is to be delayed, is put into the bottom of the array, and at the same time each element in the array is moved one step up in the array. The delayed variable is taken from the top of the array.
One can of course do the opposite as illustrated in the file **ov678_pi.m** from Exercise 8.