Master study
Systems and Control Engineering
Department of Technology
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SCE1106 Control Theory

Exercise 4

Task 1

We are in this task going to analyze controllability of a system with two identical time constants.

a)

Given a system $\dot{x} = Ax + Bu$ where

$$A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}. \tag{1}$$

Show that this system is not controllable for any $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ by using the controllability matrix criterion. Try to argue for the result by physical argumentations.

b) Consider now that the system is changed to

$$A = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}. \tag{2}$$

Investigate if the system now is controllable. Find under which conditions the system is controllable. Use the controllability matrix method.

Exercise 2

Given a process which is modelled by a state space model on observability canonical form as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_2 \end{bmatrix} u \tag{3}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \tag{4}$$

a) The system is to be controlled by a PD controller. Write down the expression for a PD controller in the laplace plane and sketch a block diagram of the closed loop feedback system.

b) Show that the control, u, can be written as the following state feedback of P control type. Assume that the setpoint, y^s, is constant, or equivalently, assume that the PD controller is implemented by assuming that $\dot{e} = -\dot{y}$.

$$u = \begin{bmatrix} -K_p & -K_pT_d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + K_py^s \tag{5}$$

Exercise 3

Given a process modelled by a state space model on canonical observability form as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u \tag{6}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \tag{7}$$

a) The process is to be controlled by a PD controller. Show that the control, u, can be written as the following state feedback controller of P type.

$$u = \begin{bmatrix} g_1 & g_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + g_3y^s \tag{8}$$

wher

$$g_1 = \frac{-K_p}{1 + K_pT_db_1} \tag{9}$$

$$g_2 = \frac{-K_pT_d}{1 + K_pT_db_1} \tag{10}$$

$$g_3 = \frac{K_p}{1 + K_pT_db_1} \tag{11}$$

Exercise 4

Given a system described by

$$y = h_p(s)u \tag{12}$$

where

$$h_p(s) = k\frac{1 - \tau s}{\tau_0^2 s^2 + 2\tau_0 \xi s + 1} \tag{13}$$

a) For which values of ξ and τ_0 do the system have oscillating behavior? Assume that the system also should be stable.

b) Find expressions for the PID controller parameters, K_p, T_i and T_d on ideal form, by the use of the Skogestad method.

Remarks

One of the reasons for Exercise 1 is to give insight in controllability analysis of systems with the controllability matrix method. In addition learn how to investigate controllability by physical insights of the system.
The point with Exercises 2 and 3 is to shpw that PD control in many cases is equivalent with state feedback. In optimal control theory which is a central part of advanced control theory, it cam be shown that optimal controllers usually are of state feedback type. A problem with the D part of the controller is the implementation of the derivative, $\dot{y}$, of the measurements, in particular when there are noise on the measurements.