Master study
Systems and Control Engineering
Department of Technology
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SCE1106 Control Theory

Exercise 9

Task 1

We are in this exercise going to design an ideal feed forward controller from a disturbance, v, and the reference, r. Given a system described by the transfer function model

$$y(s) = h_p(s)u(s) + h_d(s)v(s) \tag{1}$$

where $v(s)$ is the process disturbance, and a controller of the form

$$u(s) = h_c(s)e(s) + u_f \tag{2}$$

where $e(s) = r - y$ is the control deviation and u_f is a feed forward signal. We are assuming that the disturbance v is measured and completely known.

a) Ideally we want $y = r$. Find the ideal feedforward signal, u_f, from the demand $y = r$. Sketch a block diagram for the control system.

b) Develop an ideal feed forward controller, u_f, such that the disturbance, v, have no influence on the process output, y. Sketch a block diagram also for this situation.

Task 2 (Balchen approximation for $e^{-\tau s}$)

a) Consider given a 1st order approximation to the transport delay $e^{-\tau s}$ of the following form

$$e^{-\tau s} \approx \frac{1 - \alpha s}{1 + \alpha s} \tag{3}$$

Assume that $s = j\omega$. The phase shift to the frequency response $e^{-j\tau\omega}$ (i.e. $\angle e^{-j\tau\omega}$) is equal to $-\frac{\pi}{2}$ at the frequency $\omega_1 = \frac{\pi}{2}\frac{1}{\tau}$.

The basic for the Balchen 1st order approximation to $e^{-\tau s}$ is as follows: One want that $e^{-\tau s}$ and the approximation $\frac{1-\alpha s}{1+\alpha s}$ to be identical at the frequency ω_1.

Use this information to find the parameter α in the 1st order Balchen approximation for the time delay $e^{-\tau s}$.

b) Given a system described by a pure transport delay

$$y(s) = e^{-\tau s} u(s) \tag{4}$$

Find a state space model for (4) for a 1st order and a 2nd order Balchen approximation as found in step 1a) above.

Note that state space models is necessary for numerical simulations. This is the reason for transforming the transfer function model to time domain models.

Task 3

Given a process which can be described by a pure transport delay, i.e., with transfer function model description

$$y = h_p(s) u, \tag{5}$$

where

$$h_p(s) = e^{-\tau s}. \tag{6}$$

We are choosing a pure integral (I) controller for controlling the process, i.e.

$$u(s) = h_c(s)(r - y), \tag{7}$$

where

$$h_c(s) = \frac{1}{T_i s}. \tag{8}$$

a) We want the process output to follow a specified reference signal r. Sketch a block diagram for the closed loop system both in the s-plane domain and in the time domain.

b) Show that the control system gives zero steady state error, i.e., $y = r$ when $t \to \infty$. This can be argued from the block diagram ore via the final value theorem

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} f(s) s \tag{9}$$

c) Show that the closed loop system is stable if

$$\frac{\tau}{T_i} < \frac{\pi}{2} \tag{10}$$

by using the Bode stability criterion. Tips: find the phase crossover frequency, ω_{180}, which gives $\angle h_0(j\omega) = -180$ $[degres]$ and thereafter demanding $\mid h_0(j\omega) \mid < 1$.

d) Perform the same stability analysis by using the Balchen 1st order approximation to the time delay.

Task 4

Given a process which can be described by the following model

$$y(s) = \frac{k}{1 + T_1 s} u(s) + \frac{c}{1 + T_1 s} v(s) \tag{11}$$

where $k = 0.5$, $T_1 = 1$ and $c = 0.1$. u is the control input, v is the process disturbance and y is the process output measurement.
Show that the model can be written on state space form, i.e., find A, B, C and D in the state space model.

$$\dot{x} = Ax + Bu + Cv \tag{12}$$

$$y = Dx \tag{13}$$

Use the explicit Euler approximation to ($\dot{x}$) and write down a discrete time model.
Given a PID-controller

$$u(s) = K_p\left(\frac{1 + T_i s}{T_i s} + T_d s\right)e(s) \tag{14}$$

use an Euler approximations to the time derivatives and find a discrete time model formulation of the PID controller
The control deviation is given by

$$e = y^s - y \tag{15}$$

where y^s is the prescribed set point for the process output y.
make a MATLAB script program (*.m file) which consists of a
for i=1:N
process and controller equations
end
for simulation of the closed loop system after set point changes and high frequency noise v.
Given a process model with two time constants

$$y(s) = \frac{k}{(1 + T_1 s)(1 + T_2 s)} e^{-\tau s} u(s) \tag{16}$$

where $T_2 = 0.5$ and $\tau = 2$. Extend the simulator program above.
Use the 1st order Balchen approximation to the transport delay in the simulations.