Master study
Systems and Control Engineering
Department of Technology
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SCE4106 Model Predictive Control

Exercise: PI and MPC controller for system with time delay

Task

Consider a level system which is modeled by an integrator plus time delay as

$$y = h_p(s)(u - v). \tag{1}$$

u is the flow control input, v the outflow disturbance, and where the process transfer function is given by

$$h_p(s) = k\frac{e^{-\tau s}}{s}, \tag{2}$$

with process "gain" $k = 1$ (slope of the integrator) and time delay $\tau = 1$.
The level is initially controlled by a PI controller,

$$\begin{aligned} e &= r - y, && (3)\\ u &= h_c(s)e, && (4)\\ h_c(s) &= K_p\frac{1 + T_i s}{T_i s}, && (5) \end{aligned}$$

with an initial SIMC PI controller tuning with proportional gain, K_p, and integral time, T_i, given by

$$K_p = \frac{1}{2k\tau}, \; T_i = 8\tau. \tag{6}$$

The model is in the time domain given by

$$\dot{y}^- = k(u - v), \tag{7}$$

and a time delay

$$y = y^-(t - \tau). \tag{8}$$

It is of interest to instead use an MPC controller for the process.

a) Find a discrete time domain model for the system, including the time delay, i.a., a model of the form

$$\tilde{x}_{k+1} = \tilde{A}\tilde{x}_k + \tilde{B}u_k + \tilde{C}v_k, \tag{9}$$
$$y_k = \tilde{D}\tilde{x}_k. \tag{10}$$

First a model from input, u_k, to output before the delay, y_k^-, should be found on the form

$$x_{k+1} = Ax_k + Bu_k + Cv_k, \tag{11}$$
$$y_k^- = Dx_k, \tag{12}$$

then a model for the time delay, from the variable y_k^-, to the output, y_k, on the form

$$x_{k+1}^\tau = A^\tau x_k^\tau + B^\tau y_k^- \tag{13}$$
$$y_k = D^\tau x_k^\tau, \tag{14}$$

such that $y_k = y_{k-n_\tau}^-$ where n_τ is an integer number of delay samples. We may use

$$n_\tau = \frac{\tau}{\Delta t}, \tag{15}$$

rounded down or up to the nearest integer.

Assume a sampling interval $\Delta t = 0.1$.

In the simulations to be performed in the following tasks, Use a unit step in the reference, r, at time zero, and a unit step in the disturbance, v, at $t = 40$.

b) Simulate the system controlled by a PI controller.

c) Implement an LQ optimal controller with integral action on the form

$$u_k = u_{k-1} + G_1\Delta x_k + G_2(y_{k-1} - r_k). \tag{16}$$

d) Implement an MPC strategy with integral action, and the unconstrained case.

e) Implement an MPC strategy with integral action, with input rate of change and input amplitude constraints, respectively.

d) Compare the different control strategies with respect to:

- The Integrated Absolute Error (IEA),

$$IAE = \int_0^\infty |e(t)|dt, \tag{17}$$

- The Total Variation (TV) of the input rate of change

$$TV = \sum_{k=1}^{\infty} |u_{k+1} - u_k|, \tag{18}$$