Master study
Systems and Control Engineering
Department of Technology
Telemark University College
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SCE4006 Model Predictive Control with Implementation

Exercise 6

Task 1: MPC of non-linear chemical reactor

We are in this exercise to study a chemical reactor. The process model for the behavior from the feed rate, u, to the composition of product B, $y = x_2$, out of the reactor is given by the non-linear model

$$\dot{x_1} = -k_1 x_1 - k_3 x_1^2 + (v - x_1)u, \quad (1)$$

$$\dot{x_2} = k_1 x_1 - k_2 x_2 - x_2 u, \quad (2)$$

$$y = x_2, \quad (3)$$

where the reaction coefficients are given by $k_1 = 50$, $k_2 = 100$, $k_3 = 10$. The following steady state values for the states and control variable are given: $x_1^s = 2.5$, $x_2^s = 1$ and $u^s = 25$. The composition of product A in the feed rate into the reactor is assumed as a constant disturbance, $v = 10$.

a) Check that the steady state variables, x^s, u^s and v^s yields an equilibrium point such that $\dot{x}^s = 0$.

b) Discuss the implementation of e MPC controller in which the future control deviations are computed, i.e. $\Delta u_{k|L}^*$. Discuss the following:

- An objective criterion, J_k, in terms of future deviations $y_{k+i} - r_{k+i}$ and future control deviations Δu_{k+i-1} only, where $i = 1, \ldots, L$ and where L is the prediction horizon.
- A prediction model.

$$y_{k+1|L} = F_L^{\Delta} \Delta u_{k|L} + p_L^{\Delta} \quad (4)$$

- Process constraints $u_{min} \leq u_{k+i-1} \leq u_{max}$ and $\Delta u_{min} \leq \Delta u_{k+i-1} \leq \Delta u_{max}$. Formulate the constraints as a linear inequality

$$A \Delta u_{k|L} \leq b \quad (5)$$

- We want offset free control, i.e., $y_k = r_k$ when $k \to \infty$. Solve this be a sufficient choice of objective criterion as above.
- We also want the MPC controller to be independent of the constant disturbance, $v = 10$. How can we solve this.

- The MPC controller is based on a linearized discrete time state space model obtained from the non-linear model.

c) Implement the MPC controller in order to control the non-linear reactor model. Use explicit Euler in order to discretizise the non-linear model.

d) Compare the MPC controller with a conventional PI controller. How would you tune for a PI controller ?