

Lecture 2

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SCE1106: Control Theory part

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Revised September 2, 2016

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Contents:

1. Linearizing non-linear continuous time state space models $\dot{x} = f(x, u)$, $y = g(x, u)$ and developing linear continuous time state space models $\dot{x} = Ax + Bu$, $y = Dx + Eu$ as described in Sec. 1.12.
2. Analyzing linear systems $\dot{x} = Ax + Bu$, $y = Dx + Eu$. Stability Sec. 1.13. Eigenvalues and time constants. Ch 1.6 + 1.4.
3. Transfer function equivalent (for observable+controllable systems). Ch 1.9
4. Time delay Ch. 1.11
5. Step response of transfer function

$$y = h_p(s)u \quad (1)$$

for a 1st order system with time delay

$$h_p(s) = K \frac{e^{-\tau s}}{1 + Ts} \quad (2)$$

where K is system gain from u to y when time $t \rightarrow \infty$ equivalent by putting $s = 0$ in the above Eqs., and hence $K = h_p(s = 0)$.

The unit step response is plotted with the m-file given in the appendix of this note. See Figure 1 for illustration.

The above transfer function model is equivalent with the following linear state space model

$$\dot{x} = -\frac{1}{T}x + \frac{K}{T}u \quad (3)$$

and the output equal to x delayed $0 \leq \tau$ time instants

$$y = x(t - \tau) \quad (4)$$

i.e. $y(t) = 0$ for time instants $0 \leq t < \tau$ and $y(t) = x(t - \tau)$ when $t \geq \tau$.

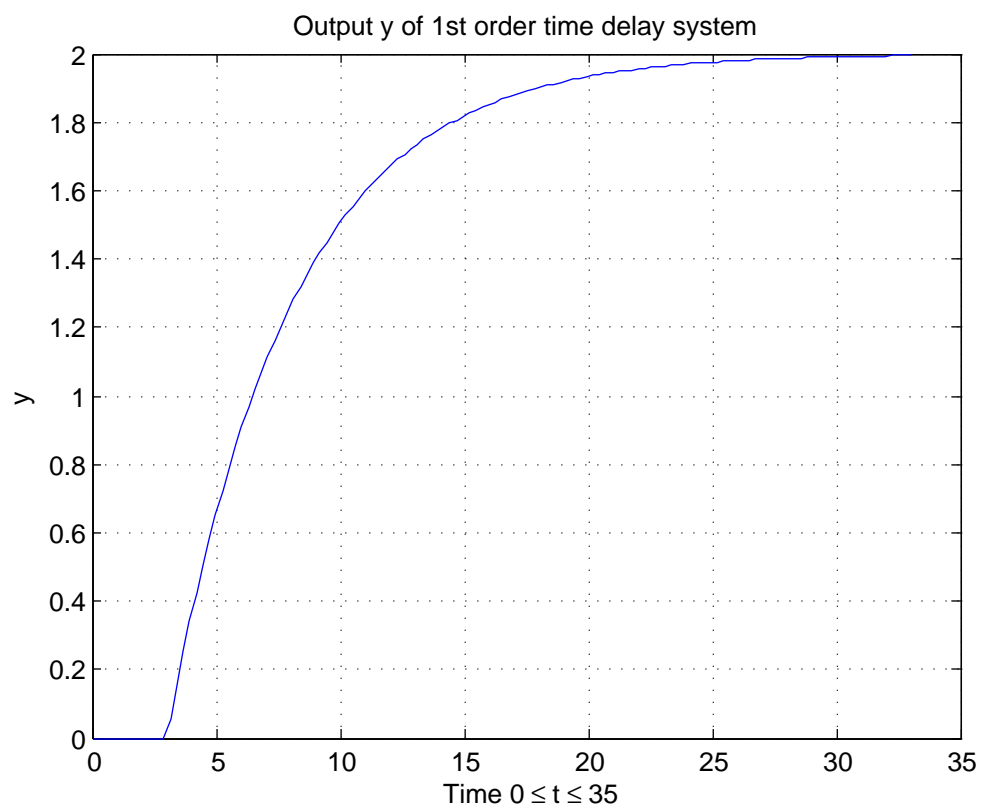
6. Poles and zeroes. Ch 1.10. The pole polynomial is equal to the characteristic equation for observable and controllable systems

$$\pi(s) = \det(sI - A) \quad (5)$$

and the poles are the roots of $\pi(s) = 0$.

7. The PI controller as described in Ch. 4.1.1
8. Designing a controller for a plant described by a 1st order plus time delay model as in Eq. (2) leading to

$$K_p = \frac{T}{K(T_c + \tau)}, \quad T_i = T \quad (6)$$



where T_c is the tuning parameter, the closed loop time constant. A rule of thumb is to chose $T_c \geq \tau$. A simple choice is simply to chose $T_c = \tau$.

This is found by solving

$$\frac{h_c(s)h_p(s)}{1 + h_c(s)h_p(s)} = \frac{y}{r} \quad (7)$$

where the best we could achieve (if the goal with the feedback loop is that the output y should be equal to the reference r)

$$\frac{y}{r} = \frac{e^{-\tau s}}{1 + T_c s}. \quad (8)$$

This means that the time delay could not be controlled away and has to be in the response and that the loop has to be dynamic with some prescribed time constant T_c .

Solving Eq. (7) for the unknown controller transfer function $h_c(s)$ gives

$$h_c(s) = \frac{1}{h_p(s)} \frac{\frac{y}{r}}{1 - \frac{y}{r}} \quad (9)$$

Using the approximation $e^{-\tau s} \approx 1 - \tau s$ we derive the controller

$$h_c(s) = \frac{T}{K(T_c + \tau)} \left(1 + \frac{1}{T s}\right) \quad (10)$$

which is a PI controller. Compare with Eq. (4.3) in the lecture notes Ch. 4.1.1 PI controller parameters is then found to be

$$K_p = \frac{T}{K(T_c + \tau)}, \quad T_i = T \quad (11)$$

We also see that the proportional gain K_p should have the same sign as the process gain K .

Appendix: MATLAB m-file for plotting unit step response

```
% Example plotting the step response of a 1st order plus time delay model
s=tf('s') % Defining s as a symbolic operator

tau=3; T=5; K=2; % Some values for time delay, time constant and gain

% Unit step response of exact transfer function
hp=K*(exp(-tau*s))/(1+T*s) % 1st order plus time delay transfer function
[Y,T]=step(hp) % Using Control Toolbox function

% Plotting the response
figure(1)
```

```
plot(T,Y), grid
title('Output y of 1st order time delay system')
xlabel('Time 0 \leq t \leq 35')
ylabel('y')
```