Systems and Control Engineering
Telemark University College
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# SCE1106 Control Theory

## Exercise 1

### Exercise 1

Given a process described by the following state space model

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -k_1 & k_2 \\ 0 & 0 & -k_3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ k_4 \end{bmatrix} u \tag{1}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -k_1 & k_2 \\ 0 & 0 & -k_3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ k_4 \end{bmatrix} \tag{2}$$

Consider given the parameters $k_1 = 1$, $k_2 = 2$, $k_3 = 4$ og $k_4 = 2$

1. Sketch a block diagram of the process model
2. Find the eigenvalues of the system matrix, $A$, and write down an eigenvalue matrix, $\Lambda$, for the system.
3. Find the eigenvectors ($m_i \ \forall \ i = 1, 2, 3$) corresponding to the eigenvalues in step 2. above.
4. Write down the eigenvector matrix

   $$M = \begin{bmatrix} m_1 & m_2 & m_3 \end{bmatrix} \tag{3}$$

   and compute the matrix inverse $M^{-1}$.
5. Check the answer by computing the matrix product

   $$M^{-1} A M = \Lambda \tag{4}$$

**Exercise 2**

Assume a process described by the following state space model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{5}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{6}$$

1. Sketch a block diagram of the system.
2. Find the transfer function from the inputs to the output of the system. I.e., find the transfer functions $h_1(s)$ and $h_2(s)$ so that

   $$y_1(s) = h_1(s)u_1(s)$$
   $$y_2(s) = h_2(s)u_2(s)$$

   and

   $$y(s) = y_1(s) + y_2(s) = h_1(s)u_1(s) + h_2(s)u_2(s). \tag{7}$$

   Check the answer by using the MATLAB functions **ss2tf** and **tf2ss** which are parts of the Control toolbox.
3. Find the eigenvalues of the system.
4. Find the steady state gain and the time constants of the system.

**Exercise 3**

A common process disturbance may be a pure sinusoid disturbance, e.g. waves and similar disturbances. Modelling of a sinusoid disturbance may be of interest in some circumstances. Assume that the output of a system is given by

$$y = dsin(\omega t + \phi) \tag{8}$$

where $d$, $\omega$ and $\phi$ are scalar parameters.

1. Transform the model in (8) to the Laplace plane (s-plane).
2. Find the state space model of a system which have the same output as in (8) by taking the Laplace model as the starting point. The model should be of the following type.

   $$\dot{x} = Ax + Bu \tag{9}$$
   $$y = Dx + Eu \tag{10}$$

   The solution of this model should be given by (8).