Master study
Systems and Control Engineering
Department of Technology
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SCE1106 Control Theory

Exercise 6

Task 1

Given a process described by the model

$$h_p(s) = \frac{e^{-2s}}{s^2 + 3s + 2} \tag{1}$$

which is to be controlled by a PI controller

$$h_c(s) = K_p \frac{1 - T_i s}{T_i s}. \tag{2}$$

a) Find the poles of the system and write the process model on the form

$$h_p(s) = k \frac{e^{-\tau s}}{(1 + T_1 s)(1 + T_2 s)} \tag{3}$$

Find the parameters k, T_1, T_2 and τ.

b) We are going to find the PI controller parameters by the Skogestad method. First use the half rule in order to find a model approximation for (1) of the form

$$h_p(s) = k \frac{1 - \tau s}{1 + T s} \tag{4}$$

Find the PI controller parameters K_p and T_i by using Skogestads method and the model approximation (4). What is the poles for the closed loop system?

c) Sketch a block diagram for the closed loop system. We want the output measurement, y, to follow a specified reference, r. Write down and plot the expressions for the *complementary sensitivity function*, $T(s)$, and *the sensitivity function*, $S(s)$, as a function of the frequency ω where $s = j\omega$. Use MATLAB!

d) Simulate the closed loop system in the time domain after a unit step response in the reference, r. Use MATLAB.

Task 2

We are in this task going to study a system described by the state space model

$$\dot{x} = Ax + Bu \quad (5)$$
$$y = Dx, \quad (6)$$

where

$$A = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; D = \begin{bmatrix} -2 & 1 \end{bmatrix}. \quad (7)$$

a) Find the poles of the open loop system!

b) Assume that the system are to be controlled by a state feedback controller of the form

$$u = -G(x_0 - x), \quad (8)$$

where x_0 is a given reference vector for the states (the state vector) x. Find a controller feedback matrix, G, such that the closed loop system got the time constants

$$T_1 = \frac{1}{4}, \quad (9)$$
$$T_2 = \frac{1}{6}. \quad (10)$$

c) Investigate if there are possibile to find a controller of the form

$$u = -g(r - y), \quad (11)$$

such that the closed loop system got the time constants $T_1 = \frac{1}{4}$ and $T_2 = \frac{1}{6}$.