Partial test
SCE4106 Model Predictive Control
Friday 5. October 2007
kl. 9.15-11.15, Rom F29

The test consists of 2 tasks.
The test counts 30 % of the final grade
in the course.
The test consists of two pages.
Aid: paper and pen.

David Di Ruscio
Tlf: 5168, Rom: B249

Master study
Systems and Control Engineering
Department of Technology
Høgskolen i Telemark

1 Partial test MPC

Task 1 (25%): MPC

Given a process described by the linear discrete time state space model

$$x_{k+1} = Ax_k + Bu_k, \tag{1}$$
$$y_k = Dx_k, \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^r$ is the control vector and $y_k \in \mathbb{R}^m$ is the output vector.
Consider also the control objective

$$J_k = \sum_{i=1}^{L}((y_{k+i} - r_{k+i})^T Q_i (y_{k+i} - r_{k+i}) + u_{k+i-1}^T P_i u_{k+i-1}) \tag{3}$$

a) Show that the control objective can be written as follows

$$J_k = (y_{k+1|L} - r_{k+1|L})^T Q (y_{k+1|L} - r_{k+1|L}) + u_{k|L}^T P u_{k|L}. \tag{4}$$

Specify the extended vectors $r_{k+1|L}$ and $y_{k+1|L}$, as well as the extended weighting matrices Q and P.

b) Show that the process model (1) and (2) can be written as a prediction model of the form,

$$y_{k+1|L} = F_L u_{k|L} + p_L. \tag{5}$$

Specify expressions for F_L and p_L.

c)

- Find the MPC optimal unconstrained future controls of the form

$$u_{k|L}^* = G(r_{k+1|L} - p_L). \tag{6}$$

  Specify the matrix G.

- Which optimal MPC control, u_k^*, is used to control the process at the present time, k?

d) Assume that we have some input amplitude constraints

$$u_{k|L}^{min} \leq u_{k|L} \leq u_{k|L}^{max} \tag{7}$$

Show that the constraints can be written as a linear inequality

$$\mathcal{A} u_{k|L} \leq b \tag{8}$$

Specify the matrix $\mathcal{A}$ and the vector b.

e) Formulate the control objective, (4), with the prediction model (5) and the constraints (8) as a Quadratic Programming (QP) problem,

Problem 1.1 (MPC QP problem)
Minimize

$$J_k = u_{k|L}^T H u_{k|L} + 2f^T u_{k|L} + J_0. \tag{9}$$

with respect to $u_{k|L}$ *subject to constraints (8).*

Specify matrix H and the vector f.

Task 2 (5%): Optimization

a) Find the minimum variables, x_1^* and x_2^*, of the quadratic function

$$J = 2x_1^2 + x_1 x_2 + x_2^2 - 12x_1 - 10x_2 \tag{10}$$