Partial test
SCE4106 Model Predictive Control
Friday 17. October 2008
kl. 10.15-12.15, Rom F29

The test consists of 2 tasks.
The test counts 30 % of the final grade
in the course.
The test consists of two pages.
Aid: paper and pen.

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Task 1 (25%): MPC

Given a process described by the linear discrete time state space model

$$x_{k+1} = Ax_k + Bu_k, \quad (1)$$
$$y_k = Dx_k, \quad (2)$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^r$ is the control vector and $y_k \in \mathbb{R}^m$ is the output vector.
Consider also the control objective

$$J_k = \sum_{i=1}^{L}((y_{k+i} - r_{k+i})^T Q_i (y_{k+i} - r_{k+i}) + u_{k+i-1}^T P_i u_{k+i-1}) \quad (3)$$

a) Shortly answer the following:

1. What is the meaning of the parameter L ?
2. Can you give some guidelines for choosing Q_i and P_i ?

b) Show that the control objective can be written as follows

$$J_k = (y_{k+1|L} - r_{k+1|L})^T Q (y_{k+1|L} - r_{k+1|L}) + u_{k|L}^T P u_{k|L}. \quad (4)$$

Specify the extended vectors $r_{k+1|L}$ and $y_{k+1|L}$, as well as the extended weighting matrices Q and P.

c) Show that the process model (1) and (2) can be written as a prediction model of the form,

$$y_{k+1|L} = F_L u_{k|L} + p_L. \quad (5)$$

Specify expressions for F_L and p_L.

d)

- Find the MPC optimal unconstrained future controls of the form

$$u_{k|L}^* = G(r_{k+1|L} - p_L). \quad (6)$$

  Specify the matrix G.
- Which optimal MPC control, u_k^*, is used to control the process at the present time, k?

e) Assume that we have some input amplitude constraints

$$u_{k|L}^{min} \leq u_{k|L} \leq u_{k|L}^{max} \tag{7}$$

and

$$\Delta u_{k|L}^{min} \leq \Delta u_{k|L} \leq \Delta u_{k|L}^{max} \tag{8}$$

Show that the constraints can be written as a linear inequality

$$\mathcal{A} u_{k|L} \leq b \tag{9}$$

Specify the matrix $\mathcal{A}$ and the vector b.

f) Formulate the control objective, (4), with the prediction model (5) and the constraints (9) as a Quadratic Programming (QP) problem, i.e.,

Problem 0.1 (MPC QP problem)
Minimize

$$J_k = u_{k|L}^T H u_{k|L} + 2f^T u_{k|L} + J_0. \tag{10}$$

with respect to $u_{k|L}$ *subject to constraints (9).*

Here You should specify the matrix H and the vector f.

Task 2 (5%): Optimization

a) Find the minimum variables, x_1^* and x_2^*, of the quadratic function

$$J(x_1, x_2) = 2(x_1 - 1)^2 + x_1 - 2 + (x_2 - 2)^2 + x_2 \tag{11}$$

b) Show that $J(x_1^*, x_2^*)$ actually is the minimum of the function $J(x_1, x_2)$?