Master study
Systems and Control Engineering
Department of Technology
Telemark University College
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SCE4006 Model Predictive Control with Implementation

Exercise 2

Task 1

Given a system described by the state space model

$$x_{k+1} = ax_k + bu_k \tag{1}$$

$$y_k = x_k \tag{2}$$

and a control criterion

$$J = \sum_{i=1}^{L} (q(r_{k+1} - y_{k+1})^2 + pu_k^2) \tag{3}$$

Here, the system parameters are given by $a = 0.7$ and $b = 0.8$. The prediction horizon is chosen as $L = 4$ in the rest of the exercise.

a) Show that the criterion can be written as

$$J = (y_{k+1|L} - r_{k+1|L})^T Q(y_{k+1|L} - r_{k+1|L}) + u_{k|L}^T P u_{k|L} \tag{4}$$

In particular define the vectors which is involved and the weighting matrices Q and P, for $L = 4$.

b) Show that the process model can be written as a Prediction Model (PM) of the form

$$y_{k+1|L} = p_L + F_L u_{k|L} \tag{5}$$

Here you should define the matrix F_L and the vector p_L. Use $L = 4$.

c) Find the optimal (MPC) control, $u_{k|L}^*$, which minimizes the control criterion subject to the PM (derived from the process model).

d) Simulate the optimal control system subject to varying weighting ratio $0 < \frac{q}{p}$ and a constant reference signal $r_k = r = 1$ for all $k \geq 0$. Compare the simulation results with the simulations in exercise 1 for which the prediction horizon was simply $L = 1$.