Master study
Systems and Control Engineering
Department of Technology
Telemark University College
DDiR, September 11, 2009

SCE4006 Model Predictive Control with Implementation

Exercise 3

Task 1

Given a system described by the state space model

$$x_{k+1} = ax_k + bu_k \tag{1}$$
$$y_k = x_k \tag{2}$$

and a control criterion

$$J = \sum_{i=1}^{L}(q(y_{k+i} - r_{k+i})^2 + R_i \Delta u_{k+i-1}^2) \tag{3}$$

where q and R_i are weighting parameters

$$\Delta u_k = u_k - u_{k-1} \tag{4}$$

is the control rate of change.
Here, the system parameters are given by $a = 0.7$ and $b = 0.8$. The prediction horizon is chosen as $L = 4$ in the rest of the exercise.

a) Show that the criterion can be written as

$$J = (y_{k+1|L} - r_{k+1|L})^T Q (y_{k+1|L} - r_{k+1|L}) + \Delta u_{k|L}^T R \Delta u_{k|L} \tag{5}$$

In particular define the vectors, $y_{k+1|L}$ and $\Delta u_{k|L}$ which is involved and the weighting matrices Q and R, for $L = 4$.

b) Show that the process model can be written as a Prediction Model (PM) of the form

$$y_{k+1|L} = p_L^{\Delta} + F_L^{\Delta}\ \Delta u_{k|L} \tag{6}$$

Here you should define the matrix F_L^{Δ} and the vector p_L^{Δ}. Use $L = 4$.

c) Find the optimal (MPC) control, $\Delta u_{k|L}^*$, which minimizes the control criterion subject to the PM (derived from the process model).

d) Simulate the optimal control system subject to varying weighting ratio $0 < \frac{q}{p}$ and a constant reference signal $r_k = r = 1$ for all $k \geq 0$. Compare the simulation results with the simulations in exercise 2 for which the prediction horizon was simply $L = 1$. Comment upon possibly steady state errors between the output y_k and the reference signal r_k.