Master study
Systems and Control Engineering
Department of Technology
Telemark University College
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SCE4006 Model Predictive Control with Implementation

Exercise 5 (Optimization)

Task 1

Consider the QP problem to minimize

$$J(x) = (x_1 - 1)^2 + (x_2 - 2.5)^2 \tag{1}$$

subject to

$$x_1 - 2x_2 + 2 \geq 0 \tag{2}$$
$$-x_1 - 2x_2 + 6 \geq 0 \tag{3}$$
$$-x_1 + 2x_2 + 2 \geq 0 \tag{4}$$
$$x_1 \geq 0 \tag{5}$$
$$x_2 \geq 0 \tag{6}$$

a) Show that the above QP problem can be formulated as to minimize the quadratic function

$$J(x) = x^T H x + 2 f^T x \tag{7}$$

subject to inequality constraints

$$Ax \leq b \tag{8}$$

Write up the matrices H, f, A and b.

b) Find the minimizing solution, x^*, numerically by using MATLAB and the **quadprog** function.

c) Check if some of the constraints are **active** at the solution.

Task 2

Consider the quadratic function

$$J(x) = x^2 + 2x + 1 \tag{9}$$

a) Find the minimum, x^*, of this function analytically.

b) Find the minimum, x^*, by using the MATLAB branch and bound function **fminbnd** when you are assuming that the solution lies in the interval $-10 \leq x \leq 10$.

Task 3

Consider the non-linear Rosenbrock function

$$J(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \tag{10}$$

a) Show that

$$x^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{11}$$

is a local minimum and that the Hessian matrix at the solution is positive definite.

b) Use the MATLAB function **fminsearch** in order to obtain the solution numerically.

Task 4

Consider the non-linear Rosenbrock function

$$J(x) = 2(x_1 - 1)^2 + x_1 - 2 + (x_2 - 2)^2 + x_2 \tag{12}$$

a) Use the MATLAB function **fminsearch** in order to obtain the solution, x^*, numerically.

b) Show that the optimal solution

$$x^* = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{13}$$

is a local minimum and that the Hessian matrix at the solution is positive definite. This means that you should check wether

$$g(x) = \frac{dJ(x)}{dx} = 0 \quad \in \; \mathbb{R}^p. \tag{14}$$

and

$$H(x^*) = \frac{dg(x)}{dx^T} = \frac{d^2J(x)}{dx^T dx} > 0 \quad \in \; \mathbb{R}^{p \times p}. \tag{15}$$

at the solution $x = x^*$ and where p is the number of unknown parameters in x.

Task 5

Perform a m-file implementation of the Newton's method for searching for the minimum of $J(x)$, i.e., implement the search

$$x_{i+1} = x_i - \alpha_i H(x_i)^{-1} g(x_i) \quad \forall \; i = 1, 2, \ldots, i_{max} \tag{16}$$

where $g(x_i)$ is the gradient and $H(x_i)$ the Hessian of the function $J(x)$. For Newtons method we have that the line search parameter is $\alpha_i = 1$. You may fix the maximum number of iterations i_{max}.

You may here use analytical expressions for the gradient and the Hessian. Take the functions in Task 3 and 4 as specific examples for the implementation.