Master study
Systems and Control Engineering
Department of Technology
Telemark University College
DDiR, November 21, 2008

SCE4006 Model Predictive Control with Implementation

Exercise 7

Task 1: MPC of a MIMO system

We are in this exercise to study a MIMO system

$$x_{k+1} = Ax_k + Bu_k, \tag{1}$$
$$y_k = Dx_k \tag{2}$$

where

$$A = \begin{bmatrix} 1.5 & 1.0 & 0.10 \\ -0.7 & 0 & 0.10 \\ 0 & 0 & 0.85 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \; D = \begin{bmatrix} 3 & 0 & -0.6 \\ 0 & 1 & 1 \end{bmatrix}. \tag{3}$$

Consider the following MPC objective

$$J_k = \sum_{k=1}^{L}((y_{k+i} - r_{k+i})^T Q(y_{k+i} - r_{k+i}) + \Delta u_{k+i-1}^T R \Delta u_{k+i-1}) \tag{4}$$

where the weighting matrices are nominally specified as

$$Q = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.03 \end{bmatrix}, \; R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{5}$$

a) The references for the two outputs should be perturbed around an initial reference

$$r = \begin{bmatrix} r^1 \\ r^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \tag{6}$$

Initially we are assuming that the system is in steady state. Compute the steady state values for the state vector, x^s, the control input, u^s such that we have $y = r$. These steady state values are to be used as initial values in the simulations.

b) Use the function **prbs1.m** and design a reference signal r_k where we are assuming a simulation horizon of $N = 250$. Assume that reference r^1 is perturbed 20% and that r^2 is perturbed 40%. Furthermore, assume that the reference signals are constant in at least $T_{min} = 30$ samples and constant maximum $T_{max} = 70$ samples

Plot the suggested reference signals

c) Discuss the implementation of an MPC controller in which the future control deviations are computed, i.e. $\Delta u^*_{k|L}$. Discuss the following:

- Try different prediction horizons L.
- A prediction model.

$$y_{k+1|L} = F_L^{\Delta} \Delta u_{k|L} + p_L^{\Delta} \tag{7}$$

- Process constraints $u_{min} \leq u_{k+i-1} \leq u_{max}$ and $\Delta u_{min} \leq \Delta u_{k+i-1} \leq \Delta u_{max}$. Formulate the constraints as a linear inequality

$$A \Delta u_{k|L} \leq b \tag{8}$$

- We want offset free control, i.e., $y_k = r_k$ when $k \to \infty$. Investigate if the MPC control strategy gives offset free control.

c) The term p_L^{Δ} is a function of the present state, x_k. Check out the strategy in which x_k is computed in terms of past inputs and outputs.