Partial test SCEV3006 Advanced Control with Implementation Wednesday October 17. 2007 kl. 9.15-11.15 + 15 minutes extra time

The test consist of : 2 tasks,
and counts 30 % of the final grade.
It is 3 pages in the test text.

Helping aid: appendix with some stuff.

October 17, 2007

Task 1 (12%): Diverse questions

a) Given a proper system

$$\dot{x} = Ax + Bu, \tag{1}$$
$$y = Dx + Eu. \tag{2}$$

- How can we use the generalized eigenvalue method in order to find the transmission zeroes of the system?
- Assume that

$$A = -1, \; B = 2, \; D = 1, \; E = -3 \tag{3}$$

  Find the transmission zeroes by using the generalized eigenvalue method.

b) Given a system, $y = H(s)u$, where the transfer matrix of the system is given by

$$H(s) = \begin{bmatrix} \frac{1}{s^2+3s+2} & -\frac{1}{s^2+3s+2} \\ \frac{s^2+s-4}{s^2+3s+2} & \frac{2s^2-s-8}{s^2+3s+2} \\ \frac{s-2}{s+1} & \frac{2s-4}{s+1} \end{bmatrix} \tag{4}$$

- Find the pole polynomial, $\pi(s)$, of the system and the system poles.
- Find the zero polynomial, $\rho(s)$, of the system and the system zeroes.

c)

- Find the general solution, $x(t)$, for the state equation (1). Assume that the initial state, $x(t_0)$, is known and that the time is, $t > t_0$.
- Use the solution in step c) above to find a discrete state space model of the form

$$x_{k+1} = \Phi x_k + \Delta u_k. \tag{5}$$

  You should write down expressions for Φ and Δ.

e) Present two methods for computing the transition matrix Φ.

Task 2 (18%) Continuous LQ optimal control

a) Assume a process modelled by the following linear state space model

$$\dot{x} = Ax + Bu, \tag{6}$$

where the initial state, $x(t_0)$, is known.

Assume that we have given an objective criterion

$$J = \frac{1}{2}x(t_1)^T S x(t_1) + \frac{1}{2}\int_{t_0}^{t_1} (x^T Q x + u^T P u)dt. \tag{7}$$

Derive an expression for the optimal control, $u(t)^* \ \forall \ t_0 \leq t \leq t_1$, for the process in (6) which minimizes the objective criterion (7).

The answer should contain a Riccati equation with boundary condition.

b) Assume that we have an objective criterion with large or infinite time horizon, i.e.,

$$J = \frac{1}{2}\int_{0}^{\infty} (x^T Q x + u^T P u)dt. \tag{8}$$

and the state space model in (6).

What is now the solution to the LQ optimal control problem? You may use the results in step 2a) above.

c) Assume now that the process is modelled by a proper state space model of the form

$$\begin{aligned} \dot{x} &= Ax + Bu, & (9) \\ y &= Dx + Eu. & (10) \end{aligned}$$

where the initial state, $x(t_0)$, is known.

Furthermore, consider the objective criterion

$$J = \frac{1}{2}x(t_1)^T S x(t_1) + \frac{1}{2}\int_{t_0}^{t_1} (y^T Q y + u^T P u)dt. \tag{11}$$

Find an expression for the optimal control for the process in (9) and (10) which minimize the objective criterion (11).

d)

- If we want to demand closed loop stability of the optimal controller in step 2b) above. What is the demands for the system matrices A and B and the weighting matrices Q and P?
- What is the minimum value of the objective criterion in step 2b) above?

Appendix

Continuous optimal control

$$\begin{aligned}
\dot{x} &= f(x,u,t) && (12)\\
J &= S(x(t_1)) + \int_{t_0}^{t_1} L(x,u,t)dt && (13)
\end{aligned}$$

The maximum principle for continuous systems

$$\begin{aligned}
H &= L + p^T f && (14)\\
\dot{p} &= -\frac{\partial H}{\partial x} && (15)\\
p(t_1) &= \frac{\partial S}{\partial x}|_{t_1} && (16)
\end{aligned}$$

Discrete optimal control

$$\begin{aligned}
x_{k+1} - x_k &= f(x_k,u_k,k) && (17)\\
J_i &= S(x_N) + \sum_{k=i}^{N-1} L(x_k,u_k,k) && (18)
\end{aligned}$$

The maximum principle for discrete time systems

$$\begin{aligned}
H_k &= L(x_k,u_k,k) + p_{k+1}^T(x_{k+1}-x_k) && (19)\\
p_{k+1} - p_k &= -\frac{\partial H_k}{\partial x_k} && (20)\\
p_N &= \frac{\partial S}{\partial x_N} && (21)
\end{aligned}$$

Derivation rules

$$\begin{aligned}
\frac{\partial}{\partial x}(Qx) &= Q^T && (22)\\
\frac{\partial}{\partial x}(x^TQx) &= Qx + Q^Tx && (23)
\end{aligned}$$