Master study
Systems and Control Engineering
Department of Technology
Telemark University College
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SCEV3006 Advanced Control with Implementation

Exercise 1

Task 1

Given a system described by the state space model

$$\dot{x}_1 = -5x_1 - 2x_2 + \beta u \quad (1)$$
$$\dot{x}_2 = -x_2 + u \quad (2)$$
$$y = x_1 + \delta x_2 \quad (3)$$

a) Sketch a block diagram for the system and write the system equation on matrix form $\dot{x} = Ax + Bu$, $y = Dx$.

b) Use the rank method of the controllability matrix in order to investigate if the system is controllable. Tips: The answer is dependent of the parameter β.

c) Use the rank method of the observability matrix in order to investigate if the system is observable. Tips: The answer is dependent of the parameter δ.

d) Transform the system to diagonal form and sketch a blook diagram of the system. use the diagonal form model in order to find fir which cases the system is controllable and observable.

e) Analysis the system for controllability and observability by using the Gramian matrix methods. Tips: Solve the Lyapunov matrices for the infinite time controllability and observability Gramian matrices, respectively.

Remark: The task is best solved by hand calculations and control computations by the use of MATLAB. Write a m-file script for the computations.

Task 2

Given a linear and autonomous system

$$\dot{x} = Ax \quad (4)$$

where the system matrix is given by: **case 1**

$$A = \begin{bmatrix} -5 & 4 & 0 & -2 \\ 0 & -5 & 6 & 0 \\ 2 & -6 & 4 & 4 \\ 1 & -2 & 0 & 4 \end{bmatrix} \tag{5}$$

Case 2

$$A = \begin{bmatrix} -3 & -1 & 0 & 0 \\ 10 & 17 & 39 & 80 \\ -12 & -32 & -79 & -161 \\ 4 & 12 & 30 & 61 \end{bmatrix} \tag{6}$$

Note that an autonomous system is only driven by nonzero initial state values, i.e., $x(t = 0) \neq 0$.

a) Find the coefficients in the characteristic polynomial by Krylovs method. Use MATLAB for the calculations.

b) Find an equivalent *controllability canonical form* for the system. use MATLAB.

c) Find the eigenvalues of the system and discuss the stability properties of the system.

Some background theory for solving Task 2 is given in Ch.1 and Ch. 2 in the Control Theory course lecture notes.