

Master study  
Systems and Control Engineering  
Department of Technology  
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## SCEV3006 Advanced Control with Implementation

### Exercise 7

#### Task 1: Discretization

Work through Exercises 26.1 and 26.2 in the Lecture Notes.

#### Task 2: Discrete LQ optimal control

Work through Example 5.1 in the Lecture notes., i.e. compute

- The solutions of the Riccati equation:  $R_0, R_1, R_2, R_3$  and  $R_4$ .
- The feedback matrices:  $G_0, G_1, G_2, G_3$  and  $G_4$ .
- the optimal control:  $u_0, u_1, u_2, u_3$  and  $u_4$ .
- The states :  $x_1, x_2, x_3, x_4$  and  $x_5$ .
- Simulate and plot the state and controls  $x_k, u_k$ .
- Plot the  $R_{2,2}(k)$  element in  $R_k$ .

It is recommended that you first compute by hand and thereby write a MATLAB script for the computations.

#### Task 3: Discrete LQ-control

Given a system

$$x_{k+1} = ax_k + bu_k, \quad (1)$$

$$y_k = x_k. \quad (2)$$

and the following objective function

$$J_i = \frac{1}{2}sy_N^2 + \frac{1}{2} \sum_{k=i}^{N-1} (qy_k^2 + pu_k^2). \quad (3)$$

a) Show that the optimal control is given by

$$u_k = g_k x_k, \quad (4)$$

$$g_k = -\frac{abr_{k+1}}{p + b^2 r_{k+1}}, \quad (5)$$

where  $r_{k+1}$  is defined by the discrete time Riccati equation

$$r_k = q + a^2 r_{k+1} - \frac{a^2 b^2 r_{k+1}^2}{p + b^2 r_{k+1}}, \quad (6)$$

$$r_N = s. \quad (7)$$

Tips: take the general matrix Riccati equation as the starting point and substitute the scalar system parameters  $a$  and  $b$  and the criterion parameters  $q$ ,  $p$  and  $s$ .

b) Use the numerical values  $a = 0.9$ ,  $b = 0.5$ ,  $q = 2$ ,  $p = 1$  and  $s = 2$ . Put  $N = 10$ ,  $i = 1$  and compute

- The optimal controls  $u_k$ ,  $k = 1, \dots, 9$ .
- the optimal outputs  $y_k$ ,  $k = 1, \dots, 10$ .
- The solution of the Riccati equation  $r_k$ ,  $k = 1, \dots, 10$ .
- The feedback parameters,  $g_k$ ,  $k = 1, \dots, 9$ .

This is most simply performed by using MATLAB. Plot the results with time.

Tips: see Example 9.1 for a solution sketch.