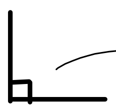
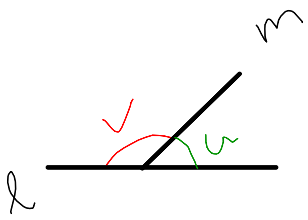
Repetisjonsforelesning og geometri

sep. 26-08.37

- Navn på ulike former

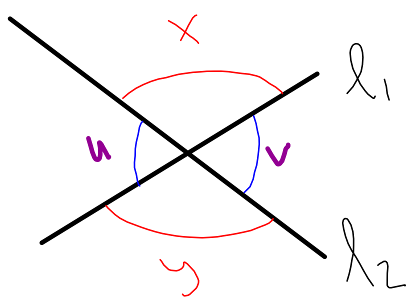


$\rightarrow 90^\circ$ $\angle C$ er rett



Nabovinkler / komplimentorvinkler

$v + u = 180^\circ$



Toppvinkler

$u = v$

$x = y$

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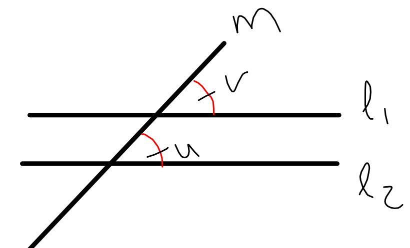


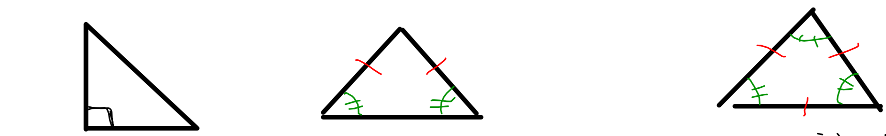
$u = v$

Samsvarende vinkler ved parallelle linjer

sep. 26-08.37

TREKANTER

$\frac{180°}{3} = 60°$



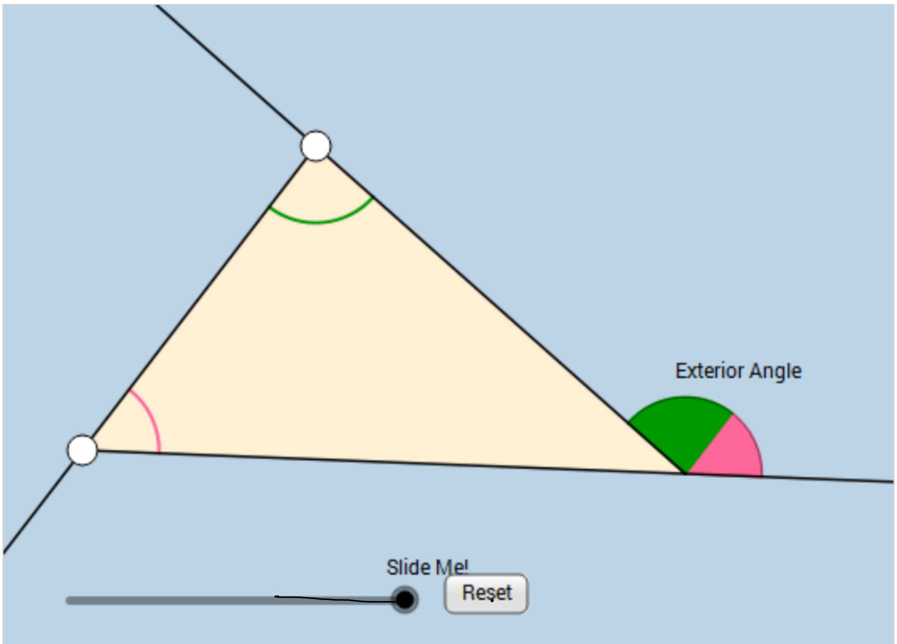
Rettvinklet trekant (En rett vinkel)

Likebeint (to sider like lange) + 2 like store vinkler

Likesidet alle sider like lange Alle vinkler like store (60°)

Summen av vinklene i en trekant er 180°

sep. 26-08.37



sep. 26-08.37

Pytagoras

Hypotenus^2 = katet^2 + katet^2

$$\boxed{\sqrt{c^2} = c}$$

kvadrat rot.

$$c^2 = a^2 + b^2$$

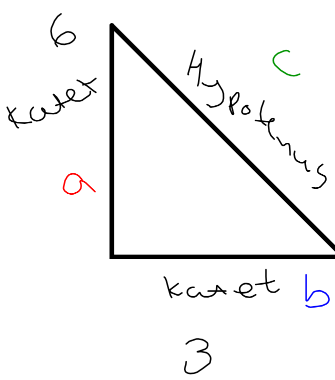
$$c^2 = 6^2 + 3^2$$

$$= 36 + 9$$

$$= 45$$

$$c = \sqrt{45}$$

$$\approx 6{,}7$$



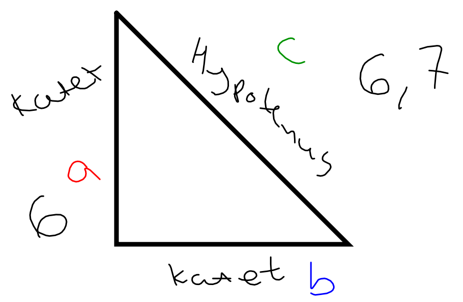
sep. 26-09.15

$c^2 = a^2 + b^2$

flytte-bytte

$a^2 + b^2 = c^2$

$b^2 = c^2 - a^2$



$b^2 = 6,7^2 - 6^2$

$= 45 - 36$

$= 9$

$b = \sqrt{9} = 3$

$6,7^2 = 6^2 + b^2$

$6,7^2 - 6^2 = b^2$

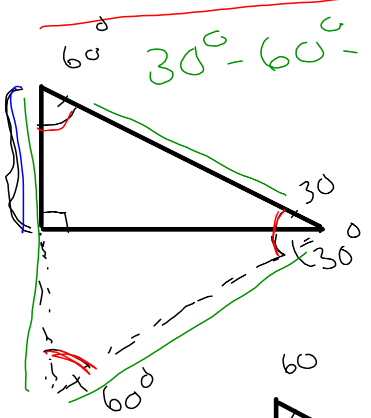
$9 = b^2 \Rightarrow b = \sqrt{9}$

$= 3$

sep. 26-09.21

Spesialtilfeller

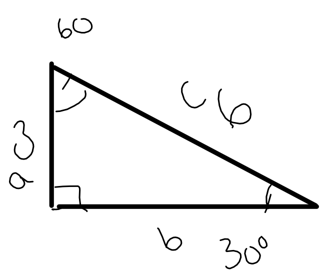
$30^\circ - 60^\circ - 90^\circ$



Da er den korteste kateten halvparten av hypotenusen

- Bevises med speiling og likesidet trekant

Finn lengdene på sidene



$c = 2 \cdot a = 2 \cdot 3 = 6$

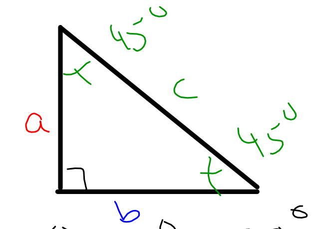
Fordi vi har en $30^\circ - 60^\circ - 90^\circ$ trekant.

$b^2 = 6^2 - 3^2 = 27$

$b = \sqrt{27} = 5,2$

sep. 26-09.25

Likebeint trekant



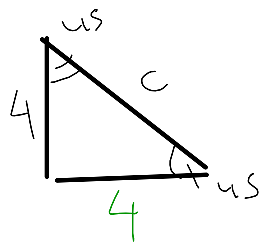
$a = b$

$c^2 = 2a^2$

$(a^2 + b^2 = a^2 + a^2 = 2a^2)$

$180° - 90° = 90°$

$\frac{90°}{2} = 45°$



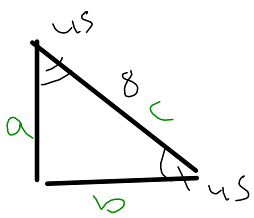
$c = 4^2 + 4^2$

$= 16 + 16$

$= 32$

$c = \sqrt{32} \approx 5,7$

sep. 26-09.32



Vi har en likebeint trekant
derfor er $a = b$.

$a^2 + b^2 = c^2$

$a^2 + a^2 = c^2$

$2a^2 = c^2$

$2a^2 = 8^2$

$a^2 = \frac{8^2}{2} = \frac{64}{2} = 32$

$a = \sqrt{32} = 5,7$, $b = \sqrt{32} = 5,7$

sep. 26-09.36

FORMLIKHET

Samme form; er enten kongruent eller forstørrelse / forminskelse av hverandre.

(Kongruent = samme størrelse og form)

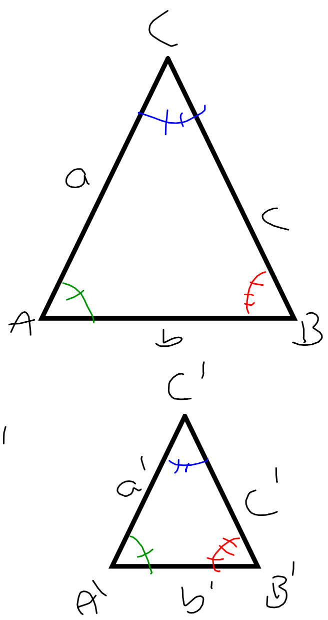
Samme vinkler og proposjonale sider

sep. 26-09.55

Formlike trekanter

1) • Alle samsvarende vinkler er like store

2) • Alle samsvarend sider er proposjonale.



1) $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$

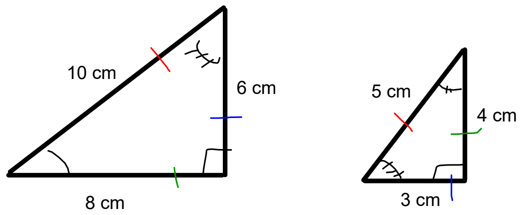
$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$ ← forholdstall

Det er nok å kjenne to vinkler

↳ fordi summen er 180.

sep. 26-09.58

Er disse formlike



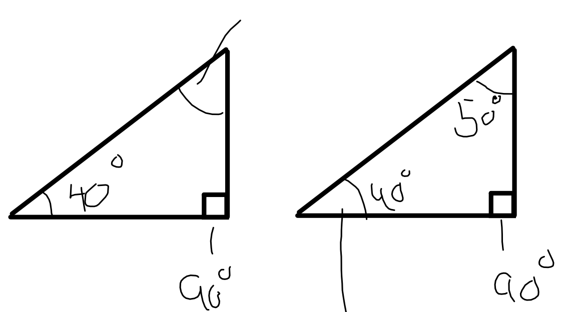
$$\frac{10}{5} = 2 \qquad \frac{6}{3} = 2 \qquad \frac{8}{4} = 2$$

De er formlike fordi sidene er proposjonale

sep. 25-19.56

Er disse formlike?

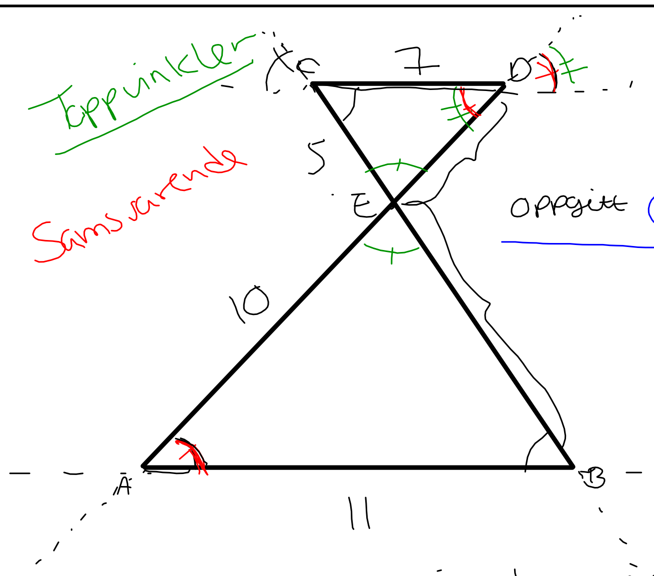
$$180^\circ - 90^\circ - 40^\circ = 50^\circ$$



Denne er formlik fordi samsvarende vinkler er like store.

$$180^\circ - 90^\circ - 50^\circ = 40^\circ$$

sep. 25-20.07



Oppgitt AB ∥ CD

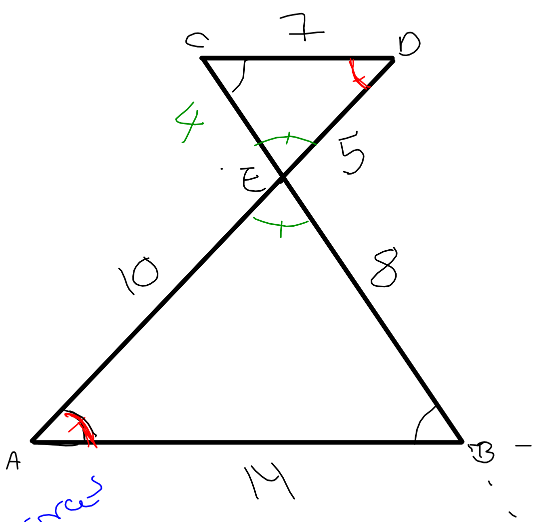
∥ betyr parallelle linjer

Er △CDE formlike med △ABE? Hvorfor?

Her bruker vi toppvinkler og samsvarende vinkler v. parallelle linje.

sep. 25-20.07

$$\frac{AE}{DE} = \frac{AB}{CD} = \frac{BE}{CE}$$



Finn lengden CE

$$\frac{BE}{CE} = \frac{AB}{CD}$$

$$\frac{8}{CE} = \frac{14}{7}$$

$$\frac{8 \cdot \cancel{CE}}{\cancel{CE}} = 2 \cdot CE$$

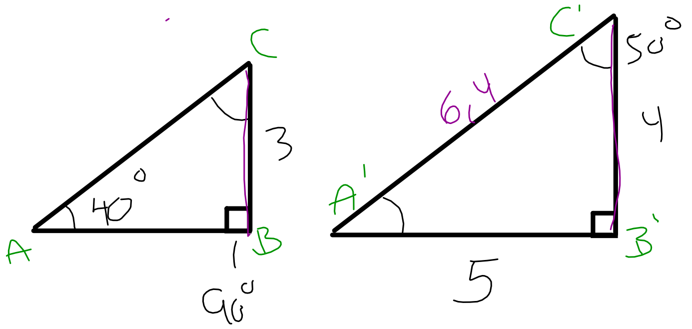
$$8 = 2 \cdot CE$$

$$CE = 8/2 = 4$$

Kan ikke bruke pytagoras fordi vi ikke kjenner vinklene

CE er 4 cm

sep. 26-10.17



Finn sidene som ikke er oppgitt.

$$\frac{BC}{B'C'} = \frac{3}{4}$$ forholdstall

$$\frac{AB}{A'B'} = \frac{3}{4}$$

$$\frac{AB}{5} = \frac{3}{4}$$

$$AB = \frac{3}{4} \cdot 5 = 3{,}75$$

Disse trekantene er formlike.

Bruker pytagoras til å finne A'C':

$$A'C'^2 = 5^2 + 4^2$$

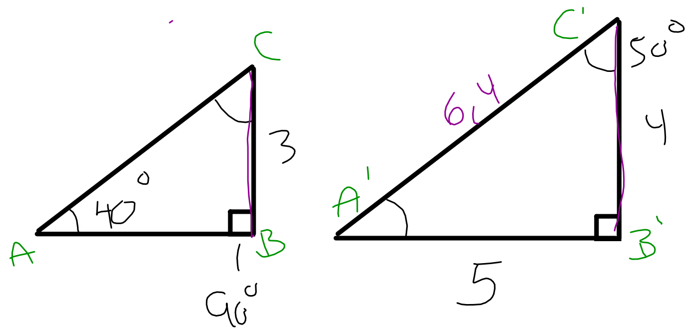
$$A'C'^2 = 25 + 16 = 41$$

$$A'C' = \sqrt{41} = 6{,}4$$

$$\frac{AC}{A'C'} = \frac{3}{4} \Rightarrow AC = \frac{3}{4} \cdot A'C'$$

$$= \frac{3}{4} \cdot 6{,}4 = 4{,}8$$

sep. 26-10.25



$AB = A'B'$ · forhold

$$\frac{AB}{A'B'}$$

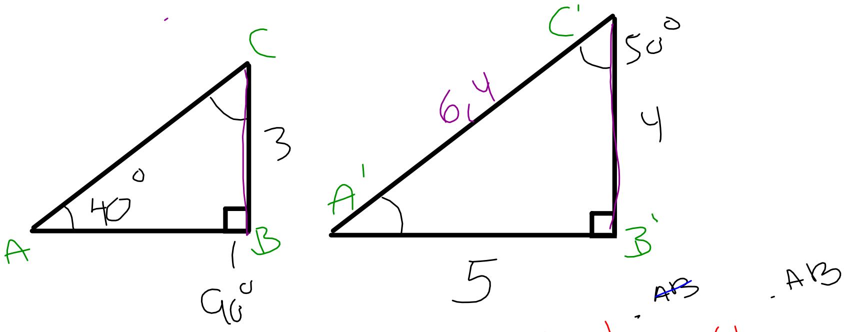
$$A'B' = AB$$

Forholdstall:

$$\frac{BC}{B'C'} = \frac{3}{4}$$ el. $$\frac{B'C'}{BC} = \frac{4}{3}$$

$$\boxed{\frac{AB}{A'B'} = \frac{BC}{B'C'}}$$

sep. 26-10.36



$$\frac{B'C'}{BC} = \frac{4}{3}$$

(forholdstall)

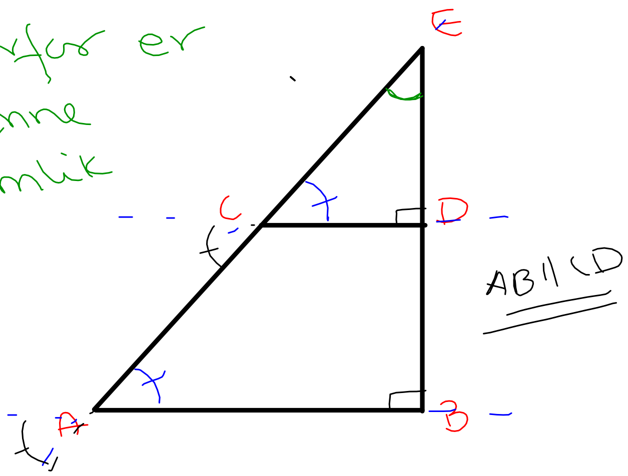
$$\frac{A'B'}{AB} = \frac{4}{3}$$

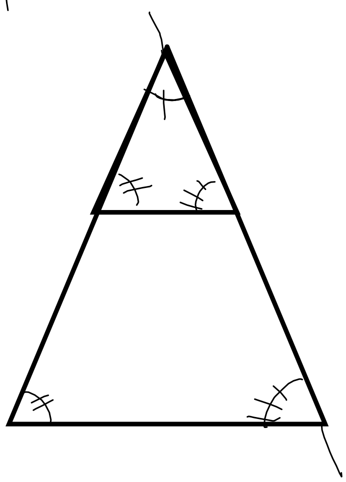
$$\frac{A'B'}{4/3} = \frac{\frac{4}{3}AB}{4/3}$$

$$\frac{A'B'}{\frac{4}{3}} = AB = A'B' \cdot \frac{3}{4}$$

sep. 26-10.36

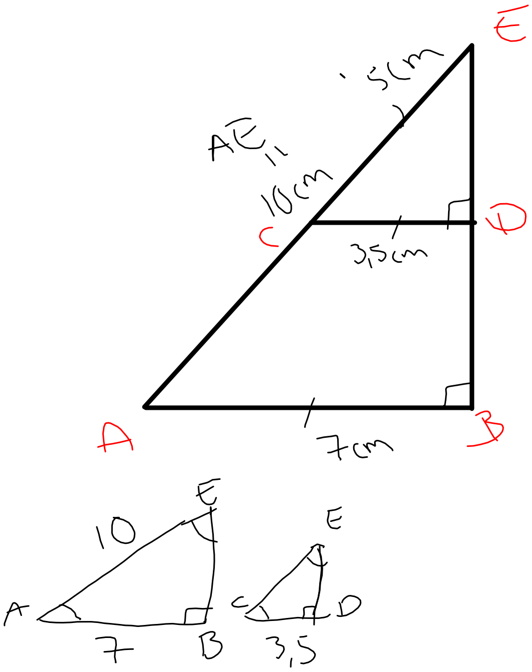
Hvorfor er denne formlik





- Bruk samsvarende vinkler ved parallelle linjer + ∠E el. 90°
- el. 90° + ∠E

sep. 25-20.12



Finn side CE?

$$\frac{AB}{CD} = \frac{AE}{CE}$$

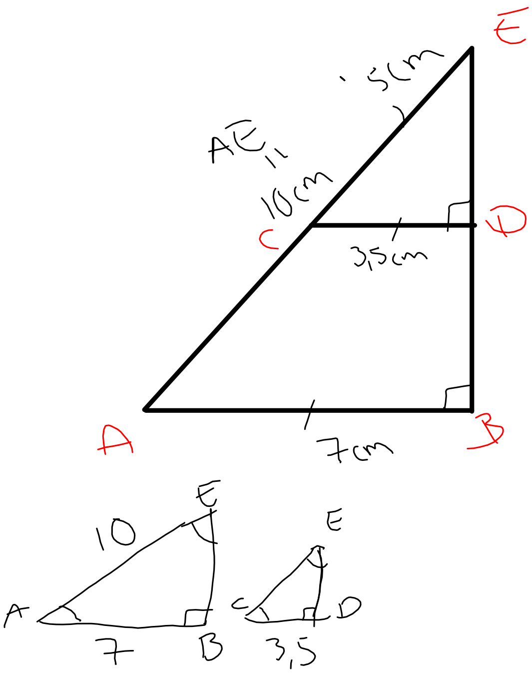
$$\frac{7}{3,5} \cdot CE = \frac{10}{CE}$$

$$2 = \frac{10}{CE}$$

$$CE = \frac{10}{2} = 5$$

$$\underline{\underline{CE = 5cm}}$$

sep. 25-20.27



Finn side $\underline{CE}$?

$$\frac{CE}{AE} = \frac{CD}{AB}$$

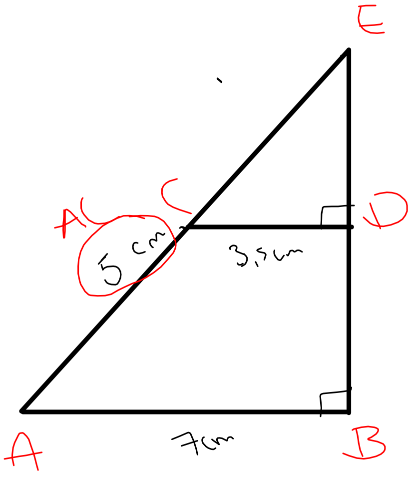
$$\frac{CE}{10} = \frac{3,5}{7}$$

$$CE = \frac{1}{2} \cdot 10$$

$$= \underline{\underline{5}}$$

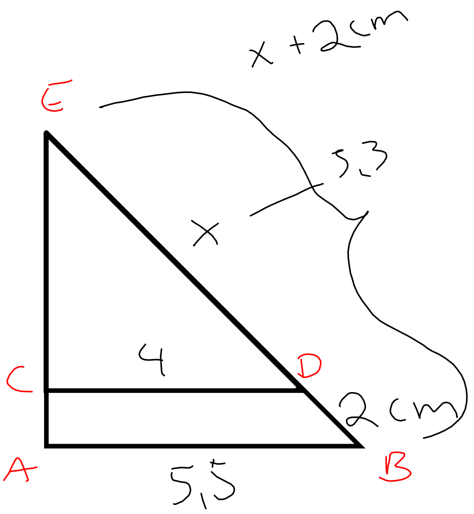
$$\underline{\underline{CE = 5cm}}$$

sep. 25-20.27



Hva er lengden AE?

sep. 25-20.28



Hva er BE?

$$\frac{BE}{DE} = \frac{5,5}{4}$$

$$\frac{x+2}{\cancel{x}} \cdot x = 1,375 \cdot x$$

$$x + 2 = 1,375x$$

$$2 = \boxed{1,375}x - x \qquad (1,375-1)x$$

$$\frac{2}{0,375} = \frac{0,375x}{0,375}$$

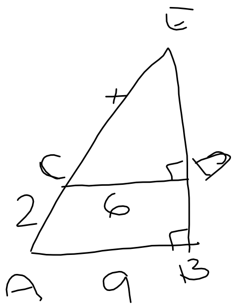
$$x = \frac{2}{0,375} = 5,3$$

$BE = 5,3 + 2$
$= 7,3$

BE er 7,3 cm

sep. 26-11.38

Finn AE:



$$\frac{AE}{CE} = \frac{AB}{CD}$$

$AE = AC + CE \Rightarrow 2+x$

$$\frac{2+x}{x} = \frac{9}{6}$$

$$2+x = \frac{9 \cdot x}{6}$$

$$6(2+x) = 9x$$

$$12 + 6x = 9x$$

$$3x = 12$$

$$x = 4$$

$AE = 2 + 4 = \underline{\underline{6}}$

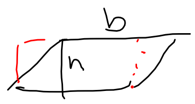
sep. 26-11.48

Areal

Rektangel

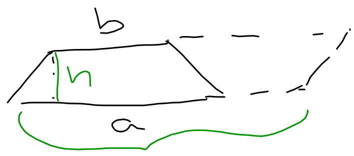
$A = l \cdot b$

Parallellogram



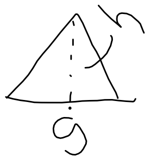
$A = h \cdot b$

trapes



$$A = \frac{(a+b)h}{2}$$

Trekanter:



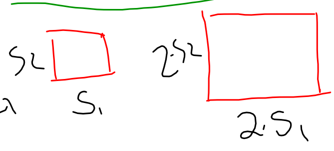
$$A = \frac{g \cdot h}{2}$$

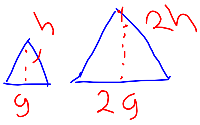
sep. 26-12.04

Areal og formlikhet.

Firkanter og trekanter:

Dobler vi lengden = firdobler vi arealet.





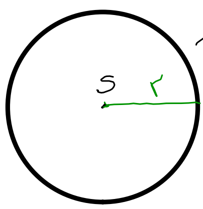
$A_1 = \frac{g \cdot h}{2}$

$A_2 = \frac{2g \cdot 2h}{2} = \frac{4gh}{2}$

$A = s_1 \cdot s_2$

$A = 2s_1 \cdot 2 \cdot s_2 = 4 s_1 \cdot s_2$

sep. 26-12.08

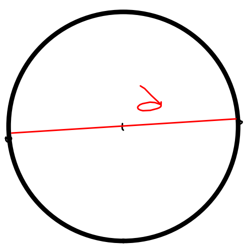


→ periferien

radius: går fra sentrum til periferien.

$A = \pi r^2$

$O = 2\pi r$



Diameter

$d = 2 \cdot r$

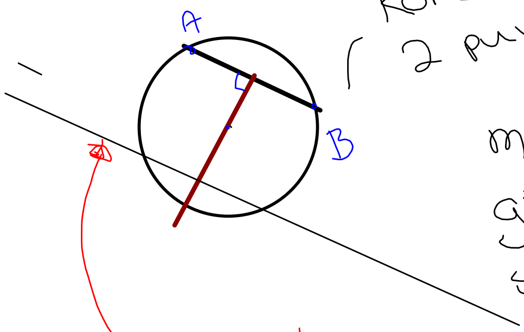
(Diameter er den lengste korden)

Konstruksjon av sirkel:

mål opp radius med passer og lag sirkel.

sep. 26-12.14

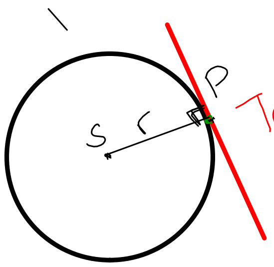
Korde: Går gjennom 2 punkter i periferien



Midtnormal til AB går alltid gjennom sentrum

Sekant
(korde som ikke har endepunkter i periferien)

sep. 26-12.19



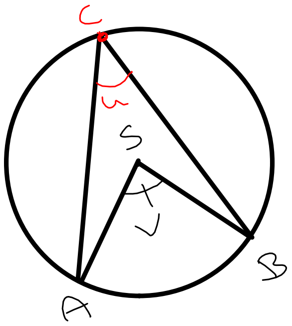
Tangent: En linje som treffer periferien i ett punkt

Normal i tangeringspunktet P går gjennom senter i sirkelen

Linja fra P til sentrum er en radius.

sep. 26-12.24

Sentral og Perefenivinkel setningen

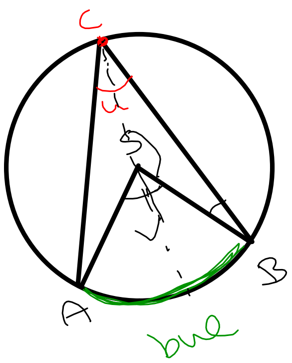


V er sentral vinkel
- topppunkt i sentrum og vinkelbein ut til perefenien.

u er en perefenivinkel.
- toppunkt i perefenien og vinkelbein ut til 2 andre punkter i perefenien

sep. 26-12.28

Sentral og Perefenivinkel setningen



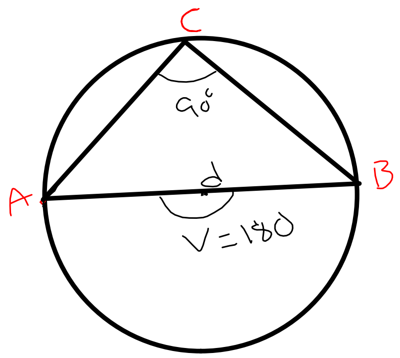
Sentralvinkelen er alltid 2 ganger så stor som periferivinkelen som spenner over den samme buen.

Vinkel ASB = 60°

hva er vinkel ACB?

sep. 26-12.28

Spesial tilfellet:

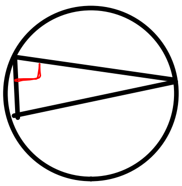


Hvor stor er sentral-vinkelen?

→ 180°

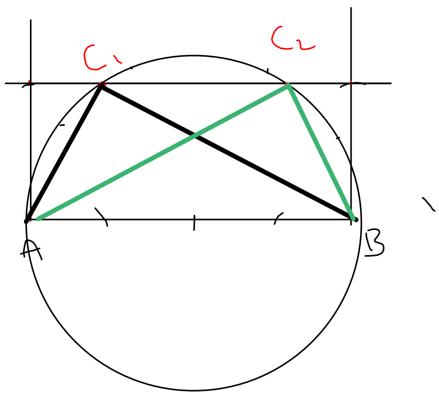
Hvor stor er ∠C (senter) sentral-

↳ 90° pga periferi vinkel setningen



↳ Thales' setning

sep. 26-12.36



$d=7 \Rightarrow r=3{,}5$

Oppgave.

AB = 7 cm

∠C er rett

Avstand fra C til AB er 3 cm

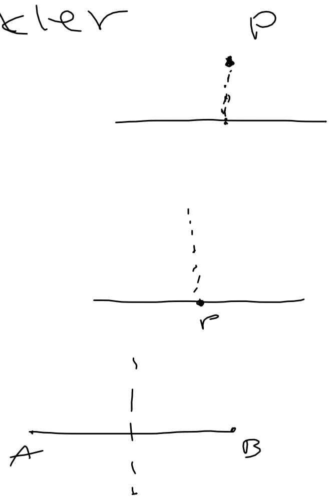
↳ Konstruer denne

Finnes det flere løsninger?

sep. 26-12.40

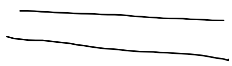
Konstruksjon

- 90°, 60°
+ halvering av vinkler
- Normaler (90°)
  - fra punk til linje
  - 1 punkt på linje
  - Midtnormal

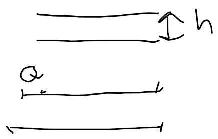


sep. 26-12.49

- Parallelle linjer



- Gitt avstand
- Gitt et punkt



Linjer som har konstant avstand og derfor ikke krysser hverandre.

$l_1 \parallel l_2$

sep. 26-12.52