

Two degrees of freedom (2-DOF) helicopter

1. Process description

The 2-DOF helicopter unit developed at USN is a dynamic system with multiple inputs and multiple outputs (MIMO). Figure 1 shows the schematic of such a unit with the side view and the top view. It consists of two propellers (pitch and yaw) driven by motors.

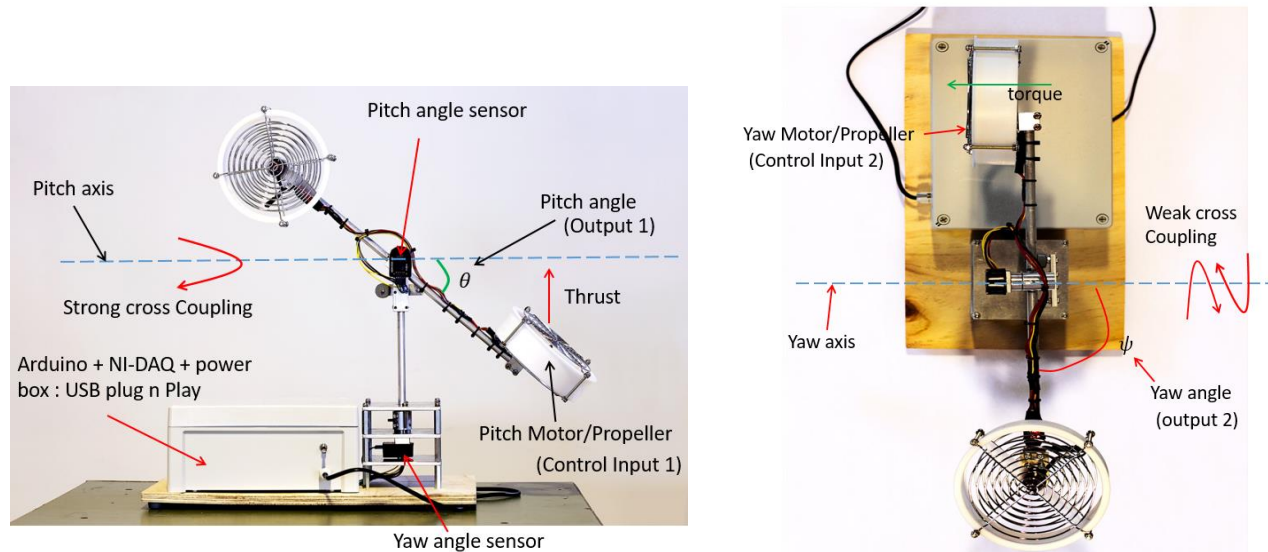


Figure 1: 2-DOF helicopter unit at USN (side view and top view)

The unit has two inputs: (a) voltage to the front or pitch motor/propeller system, and (b) voltage to the back or yaw motor/propeller system. When voltage is applied to the pitch motor, the pitch propeller rotates and it generates thrust, and the helicopter lifts up. Thus voltage to the pitch motor/propeller control the elevation (or pitch) of the helicopter nose about the pitch axis. When voltage is applied to the yaw motor, the yaw propeller rotates and it generates torque in anti-clockwise direction, and the helicopter rotates about the yaw axis. The angle between the pitch axis and the helicopter body axis is called the pitch angle. The angle between the yaw axis and the helicopter body axis is called the yaw angle. The pitch and the yaw angles are measured by using the angle sensors as shown in Figure 1. Thus, these are the two outputs of the system which are measurable.

Let us define the following:

Inputs:

V_{mp} := voltage applied to the pitch motor

V_{my} := voltage applied to the yaw motor

Outputs:

θ := pitch angle

$\psi :=$ yaw angle

The process is a cross-coupled MIMO system. When sufficient voltage is applied to the front motor, the helicopter not only pitches up but it also starts to rotate at the same time i.e. the input V_{mp} affects both outputs θ and ψ . Similarly, when sufficient voltage is applied to the back motor, the helicopter rotates in the anti-clockwise direction and at the same time, it also changes its pitch a little i.e. the input V_{my} affects both outputs θ and ψ . The effect of V_{mp} on ψ is very strong denoted by strong cross-coupling in Figure 1, while the effect of V_{my} on θ is weak denoted by weak cross-coupling.

The system can be described with four states:

$\theta :=$ pitch angle

$\psi :=$ yaw angle

$\omega_\theta :=$ pitch angular velocity

$\omega_\psi :=$ yaw angular velocity

The block diagram of the system showing the inputs, outputs and the states is shown in Figure 2.

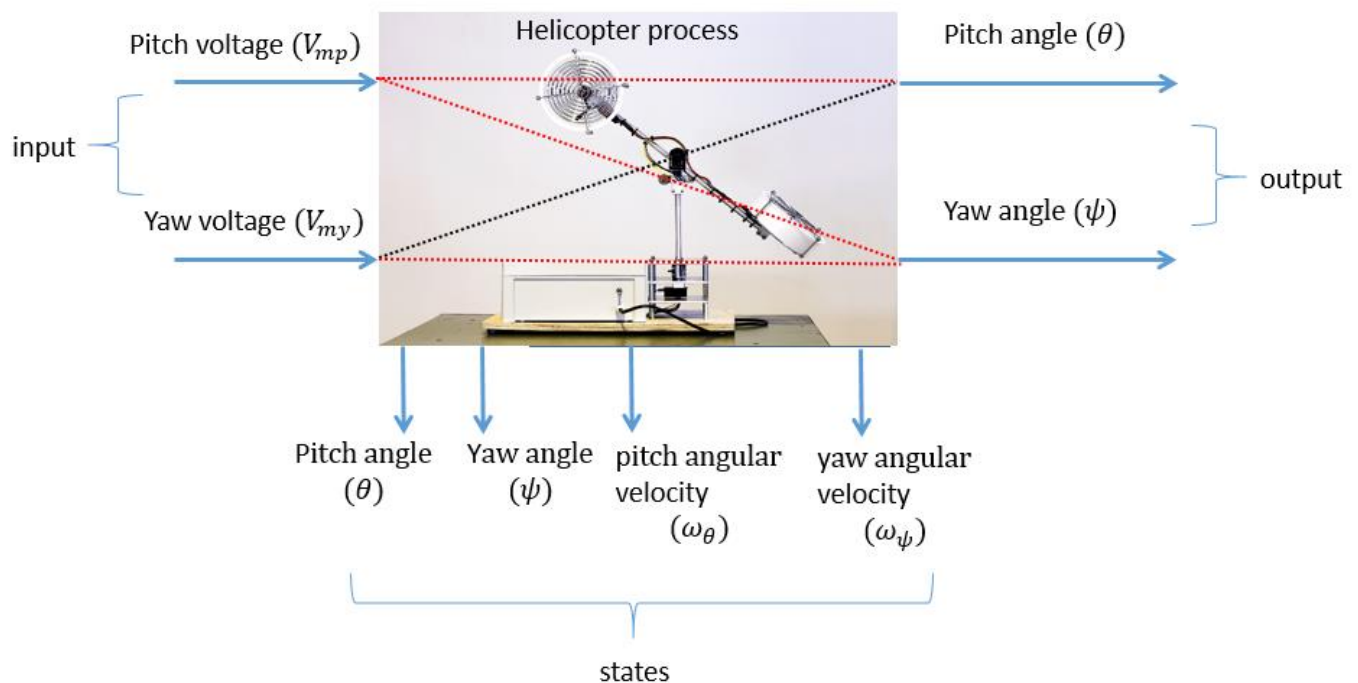


Figure 2: Block diagram of the system showing inputs, outputs and states

2. Mathematical model of the process

The dynamics of the 2-DOF helicopter unit is modelled using Newton's laws of motion and Euler-Lagrange equations. The model is described by a set of four ordinary differential equations (ODEs).

$$\frac{d\theta}{dt} = \omega_\theta \quad (1)$$

$$\frac{d\psi}{dt} = \omega_\psi \quad (2)$$

$$\frac{d\omega_\theta}{dt} = \frac{K_{pp}V_{mp}}{J_{eq,p} + m_{heli} l_{cm}^2} - \frac{K_{py}V_{my}}{J_{eq,p} + m_{heli} l_{cm}^2} - \frac{B_p \omega_\theta + m_{heli} \omega_\psi^2 \sin(\theta) l_{cm}^2 \cos(\theta) + m_{heli} g \cos(\theta) l_{cm}}{J_{eq,p} + m_{heli} l_{cm}^2} \quad (3)$$

$$\frac{d\omega_\psi}{dt} = \frac{K_{yp}V_{mp}}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} - \frac{K_{yy}V_{my}}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} + \frac{2m_{heli} \omega_\psi \sin(\theta) l_{cm}^2 \cos(\theta) \omega_\theta}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} - \frac{B_y \omega_\psi}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} \quad (4)$$

Some nonlinearities present in the real helicopter units:

Let us define,

V_{mp}^{dead} :

= minimum voltage on pitch motor required to just lift up the helicopter (≈ 1.1 [V] found experimentally)

V_{my}^{dead} :

= minimum voltage on yaw motor required to just rotate the helicopter (≈ 0.5 [V] found experimentally)

If the yaw voltage is less than V_{my}^{dead} , then $V_{my} = 0$ in Equations (3) and (4), i.e.

For $V_{my} \leq V_{my}^{dead}$

$$V_{my} = 0$$

Further more, if the pitch voltage is less than V_{mp}^{dead} , then it has no effect on the yaw angle ψ . Thus, the first term of Equation (4) is set to zero, i.e.

For $0 \leq V_{mp} \leq V_{mp}^{dead}$,

$$\frac{d\omega_\psi}{dt} = - \frac{K_{yy}V_{my}}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} + \frac{2m_{heli} \omega_\psi \sin(\theta) l_{cm}^2 \cos(\theta) \omega_\theta}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} - \frac{B_y \omega_\psi}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} \quad (5)$$

You do not necessarily have to worry and implement these nonlinearities when simulating the system i.e. you can use equations 1 - 4 for openloop simulation of the helicopter system. The feedback controller should be able to cope up with such nonlinearities.

There are altogether 11 parameters of the system. l_{cm} is a parameter which denotes the distance of the pivot point from the center of mass as shown in Figure 3. The helicopter rotates freely (up/down and clock/anti clock wise) about the pivot point. Center of mass is the point at which the gravitational force act on the system.

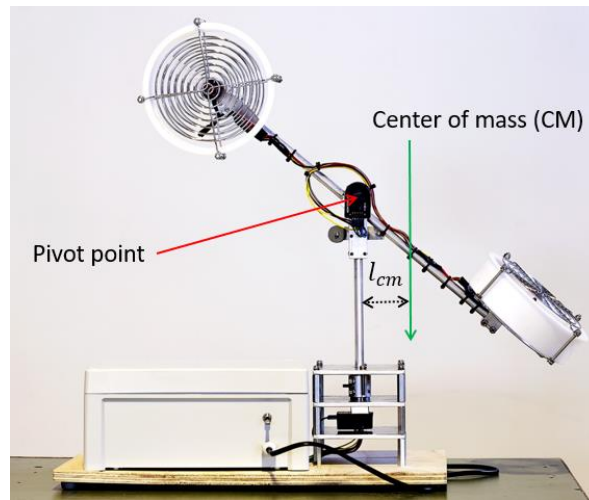


Figure 3: Illustration of center of mass and pivot point

All the helicopter units at USN are fastened and screwed in a way that $l_{cm} = 1.5 [cm]$. For a fixed value of l_{cm} and m_{heli} , the moment of inertia can be calculated and have been listen in Table 1.

Table 1: Parameters of the system: part 1

Parameter	Description	Value
l_{cm}	Distance between the pivot point and the center of mass of helicopter	0.015 [m]
m_{heli}	Total moving mass of the helicopter	0.479 [kg]
$J_{eq,p}$	Moment of inertia about the pitch axis	0.0172 [kg-m ²]
$J_{eq,y}$	Moment of inertia about the yaw axis	0.0210 [kg-m ²]
g	Acceleration due to gravity on planet earth	9.81 [m-s ⁻²]

The remaining 6 parameters of the system namely, $K_{pp}, K_{yy}, K_{py}, K_{yp}, B_p$ and B_y are slightly different for each unit of the helicopter at USN. For one of the unit, Table 2 lists the values of these parameters. You can use the parameters listed in Table 2 for simulations. But when you apply it to a real helicopter unit (the one that is assigned to you), make sure that you fine tune the parameters if necessary.

Table 2. Parameters of the system: part2

Parameter	Description	Value
K_{pp}	Torque constant on pitch axis from pitch motor/propeller	0.0556 [Nm/V]
K_{yy}	Torque constant on yaw axis from yaw motor/propeller	0.21084 [Nm/V]
K_{py}	Torque constant on pitch axis from yaw motor/propeller	0.005 [Nm/V]
K_{yp}	Torque constant on yaw axis from pitch motor/propeller	0.15 [Nm/V]

B_p	Damping friction factor about pitch axis	0.01	[N/V]
B_y	Damping friction factor about yaw axis	0.08	[N/V]

3. Data Acquisition and voltage conversion

The rectangular white box attached to the base plate in Figure 1 is used for data acquisition. Inside this box, a data acquisition device from National Instrument, NI-USB-6001 is present. This is used for reading the pitch and yaw angles of the helicopter, and for sending the control voltages to the pitch and yaw motors. There are also other components like the power supply unit and Arduino inside the white box. Reading and writing signals is via USB Plug n Play feature. There is a USB cable connected to the data acquisition box. This USB cable can be directly connected to a computer for acquiring and sending data from/to the helicopter.

3.1 Reading pitch and yaw angles of the helicopter

The analogue input channel of the data acquisition can be used to read the pitch and yaw angles of the helicopter. The angles are read as voltage signals and NOT directly as angles in radians. Thus, these voltage signals have to be converted to actual angles in radians using linear relationship.

Channel ai1 : Pitch angle

Read as reference single ended voltage in the range [0 5] volts

It corresponds to angles in the range [-0.7854 0.7854] radians (θ)

Channel ai0 : Yaw angle

Read as reference single ended voltage in the range [0 5] volts

It corresponds to angles in the range [0 π] radians (ψ)

3.2 Applying voltages to the pitch and yaw motors

The analog output channel of the data acquisition can be used to apply the voltages to the pitch and yaw motors.

Channel ao1 : voltage to the pitch motor or front motor, (V_{mp})

Sent as reference single ended voltage

Careful: The range of V_{mp} should be [0 5] volts. Applying voltages outside of this range might damage the pitch motor

Channel ao0 : voltage to the yaw motor or tail motor, (V_{my})

Sent as reference single ended voltage

Careful: The range of V_{my} should be [0 5] volts. Applying voltages outside of this range might damage the yaw motor

4. Correct way of starting up the helicopter

Before you turn ON the power switch button located at the side of the data acquisition box (the white rectangular box), the helicopter should be placed at the HOME position. The home position is when $\theta = -45^\circ$ or -0.7854 [rad], and $\psi = 0^\circ$ or 0 [rad]. This is shown in Figure 4.

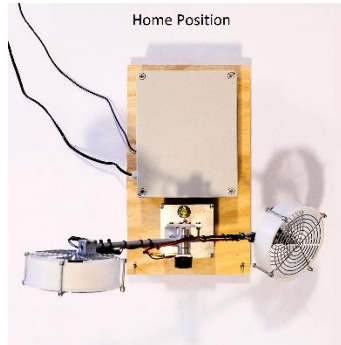


Figure 4: The home position of the helicopter unit

Home position: With white data acquisition box facing away from you, rotate the helicopter anti-clockwise to its farthest end and let it freely rest.

After the helicopter is at the home position, turn on the power switch button. Wait for beeping sound from both the pitch motor and the tail motor (you will hear two beeping sounds, sometimes together and other times with a little delay between the two sounds). After the beeping sound, the system is ready to be used.

Note: Only after the beeping sound is finished, you should “Run” your simulation program in Simulink.