LabWork 1

IIA 4117 Model Predictive Control

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1. Process description

The lab work is related to a drilling operation. A simplified diagram of an oil well drilling system is shown in Figure 1.

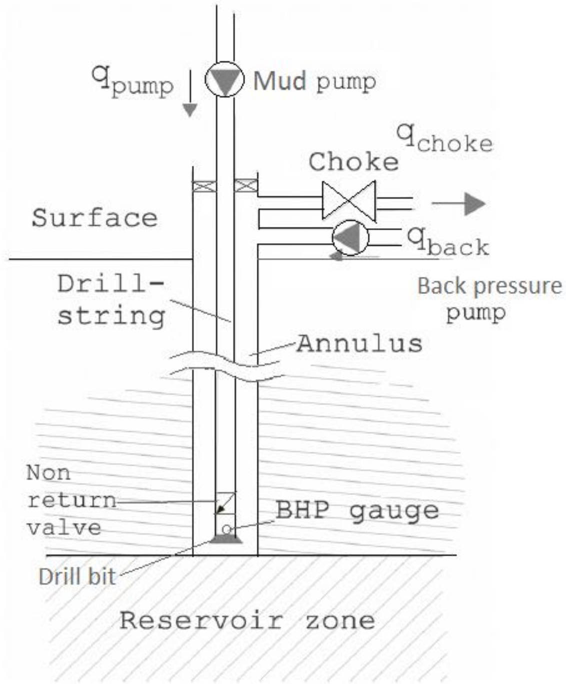

Figure 1. Simplified diagram of an oil well drilling system. [1]

Oil well drilling is performed to create wells that extend several kilometers into the ground (in offshore drilling, below the sea bed). Drill bits are attached to the end of the drill string. The drill bits are rotated using a drive system at the top side, and they cut out material from the surface (rock, soil etc.) being drilled. There is a drill fluid circulation system where the mud pump is used to inject the drill fluid (also known as drill mud) down through the drill string. The mud then flows into the annulus through the drill bit and returns upwards. Finally the mud flows out of the wellbore through the choke valve and then to the mud pit (not shown in Figure 1) normally through open flow channels. The drill bit contains a non-return valve or check valve which prevents the backward flow of fluid into the drill string from the annulus. One important function of the drill fluid is to maintain a certain pressure gradient in the annulus along the length of the well. The drill fluid is also used to transport the cuttings from the drill bit at the bottom of the well up to the surface [1], [2]. After the fluid exits the annulus, it is cleaned (sand, rock chips and other particles are removed) and recycled back as the input to the mud pump. The bottom hole pressure (BHP) gauge measures the pressure in the annulus at the bottom of the well bore.

During drilling, it is necessary to keep the bottom hole pressure of the annulus within the available drilling window. This drilling window is defined by the fracture pressure as the upper limit and the collapse pressure as the lower limit. If the bottom hole pressure is greater than the fracture pressure, the mud weight will fracture the rock and the drilling fluid will enter into the well pores. Some of the drilling fluid may block the well perforations (well openings) and productivity from the well will be lost when it is set into operation. If the bottom hole pressure is smaller than the collapse pressure, the walls of the well bore will fall off or collapse. The drill string may be stuck and a new well may have to be drilled again. If the bottom hole pressure is smaller than the reservoir pressure while drilling in the reservoir zone, some of the reservoir fluid will flow into the well. If this reservoir influx is not controlled, oil well blow out might occur leading to environmental damage and possible loss of lives. The well may be lost and this significantly increases the cost for drilling. Therefore, control of pressure for a proper management of the drilling operation is very important.

The pressure at the bottom of the well is a directly associated with the amount of drill fluid present in the annulus. More (less) is the drill fluid in the annulus, more (less) is the hydrostatic pressure due to liquid column and hence more (less) is the pressure at the bottom of the well. The amount of drill fluid present in the annulus can be changed by changing the drill fluid flow through the choke valve. At the same time, it can also be changed by using the back pressure pump to pump in more drill fluid into the annulus.

In the group project work of this course (later on), students will be asked to design a nonlinear model predictive controller for controlling the pressure at the bottom hole of the annulus within its margins. The choke valve opening and the flow rate through the back pressure pump are used as control input variables that are manipulated to keep the bottom hole pressure within the drilling window. But for now, in this labwork, students should setup the openloop simulation environment, i.e. the students should simulate the model of the process and study dynamic behavior of the system.

2. Model of the drilling process

A simple model of the drilling operation that is readily used for designing control systems and for estimation can be found in the literature as in [2], [3] and [4]. A similar model has been used for the project work with some modifications. The drill string and the annulus are considered as two separate control volumes that are connected through the drill bit's non return valve as shown in Figure 2.

The model is made using mass balances and momentum balances. Details about the development of the model not considered here, and only the final model equations are presented below.

Assume that the vertical depth of the drill string and the annulus are equal i.e. $l_{bit} = l_w = L$. From the mass balance in the drill string, dynamics of the pressure at the well head of the drill string, P_p (pressure downstream the mud pump) can be written as,

$$\frac{dP_p}{dt} = \frac{\beta_d}{A_d L}(q_{pump} - q_{bit}) \tag{1}$$

Here, q_{pump} is the volumetric flow rate of the drill mud through the mud pump, q_{bit} is the volumetric flow rate of the drill mud into the annulus through the drill bit, β_d is the bulk modulus at the drill string and A_d is the cross sectional area of the drill string.

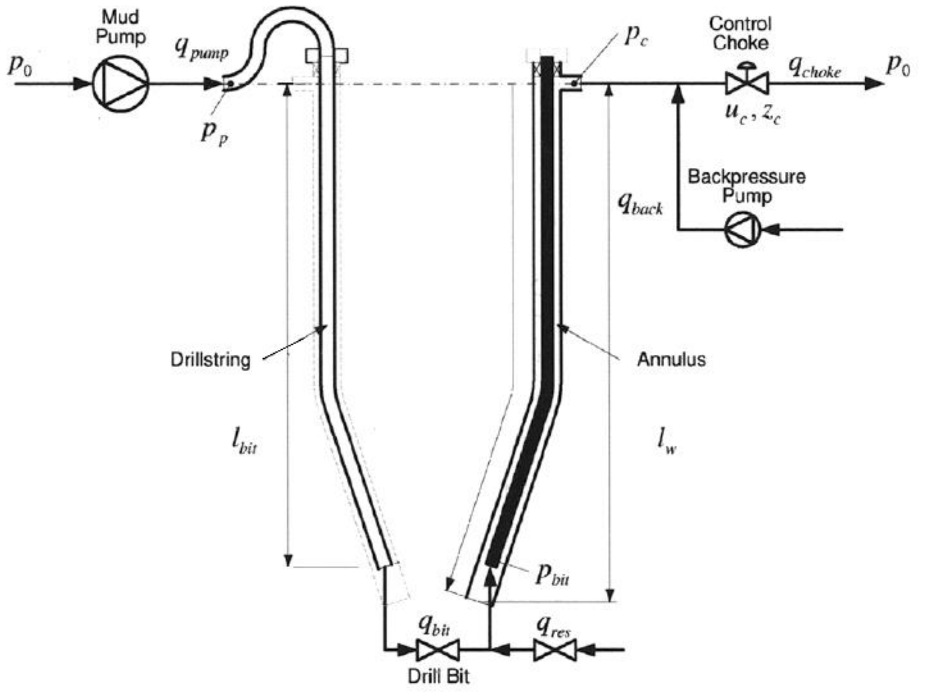

Figure 2. Drilling system modeled as two control volumes [2]

From the momentum balance, the dynamics of q_{bit} is written as,

$$\frac{dq_{bit}}{dt} = \frac{A_d}{\rho_l L}(P_p + \rho_l gL - \Delta P_f^d - P_{bit}) \tag{2}$$

Here, ρ_l is the density of the drill mud which is assumed to be constant at all sections of the drill string, g is the acceleration due to gravity, ΔP_f^d is the pressure loss in the drill string due to friction and P_{bit} is the bottom hole pressure in the annulus. The drill bit contains a non return valve to prevent the flow of fluid from the annulus back into the drill string. When $q_{bit} = 0$, the equation for the dynamics of q_{bit} reduces to,

$$\frac{dq_{bit}}{dt} = \max\left\{0, \frac{A_d}{\rho_l L}\left(P_p + \rho_l gL - \Delta P_f^d - P_{bit}\right)\right\} \tag{3}$$

From mass balance for the annulus, the differential equation for the well head pressure at the annulus (P_c) is given by,

$$\frac{dP_c}{dt} = \frac{\beta_a}{A_a L}(q_{bit} + q_{res} + q_{back} - q_{choke}) \tag{4}$$

Here, β_a is the bulk modulus at the annulus, A_a is the cross sectional area of the annulus, q_{res} is the volumetric flow rate of the reservoir fluid (mixture of oil and water, production of gas from the reservoir is not considered) and q_{back} is the volumetric flow rate of the back pressure pump which is used to inject drill fluid back into the annulus to change P_{bit} in addition to controlling the choke valve. q_{choke} is the volumetric flow rate of fluid through the choke valve.

The bottom hole pressure at the annulus P_{bit}, is given by,

$$P_{bit} = P_c + \rho_{mix} g L + \Delta P_f^a \tag{5}$$

Here, ρ_{mix} is the density of fluid in the annulus (mixture of drill fluid and reservoir fluid) and ΔP_f^a is the pressure loss due to friction in the annulus. The equation for ρ_{mix} is given by,

$$\rho_{mix} = \rho_w W_c + (1 - W_c)\rho_l \tag{6}$$

Here, ρ_w is the density of water and W_c is the water cut of the reservoir fluid. Water cut denotes the amount of water present in a unit volume of the reservoir fluid. For e.g. $W_c = 0.1$ denotes that 10% of the reservoir fluid is water and 90% of the reservoir fluid is oil. Remember that when there is no flow of reservoir fluid into the annulus i.e. when $q_{res} = 0$, then $W_c = 0$ which means that under such condition $\rho_{mix} = \rho_l$.

The volumetric flow rate of the reservoir fluid flowing into the annulus (q_{res}) is given by the *Productivity Index (PI)* model [5] of the well as,

$$q_{res} = max\{PI(P_{res} - P_{bit}), 0\} \tag{7}$$

Here, P_{res} is the pressure of the reservoir. There will be no flow of drill mud from the annulus into the reservoir pores until $P_{bit} < P_{frac}$, where P_{frac} is the fracture pressure (the upper limit of the drilling window). The model of the fluid flow when $P_{bit} > P_{frac}$ (i.e. when fracture occurs) has not been modeled (we try to avoid this from happening by using NMPC). Also Equation 7 makes it sure that when $P_{bit} > P_{res}$, that there is no flow of drill mud into the reservoir.

The volumetric flow rate of fluid through the choke valve (q_{choke}) can be expressed using the standard flow equation ANSI/ISA S75.01 developed by Instrument Society of America [6] as,

$$q_{choke} = \bar{N}_6 Z_c(u_c) \sqrt{\frac{\max(P_c - P_0, 0)}{\rho_{mix}}} \tag{8}$$

Here, $\bar{N}_6 = N_6/(3600\sqrt{10^5})$ with $N_6 = 27.3$. The valve characteristics as a function of its opening, $Z_c(u_c)$ is modeled using three linear equations by fitting the data supplied by the choke supplier as,

$$Z_c(u_c) = \begin{cases} 0 & u_c < 5 \\ 0.111u_c - 0.556 & 5 \leq u_c < 50 \\ 0.5u_c - 20 & u_c \geq 50 \end{cases} \tag{9}$$

Make sure that $Z_c(u_c) \geq 0$. Finally, the equation representing the total height of the well bore drilled is given by,

$$\frac{dL}{dt} = v_{pen} \tag{10}$$

where v_{pen} is the rate of penetration or rate of drilling. You can use $v_{pen} = 0$ for this labwork.

Pressure loss due to friction:

The pressure loss due to the flow of fluid through a pipeline in general, can be calculated by using Darcy-Weisbach formula [7] as,

$$\Delta P_f^x = \frac{f_d L \rho v^2}{2D_h} \tag{11}$$

Here, $x = d\ (for\ drill\ string)$ & $a\ (for\ annulus)$, ρ is the density of the fluid in the pipeline, v is the velocity of the fluid in the pipeline and D_h is the hydraulic diameter of the pipeline. The velocity can be calculated as $v = q/A$ with q being the volumetric flow rate and A being the cross sectional area of the pipeline.

The Darcy friction factor f_d can be evaluated using Coolebrook-White equation [7] as,

$$\frac{1}{\sqrt{f_d}} = -2\log_{10}\left(\frac{\epsilon}{3.7D_h} + \frac{2.51}{N_{Re}\sqrt{f_d}}\right) \tag{12}$$

Here, ϵ/D_h is the relative roughness of the pipe and N_{Re} is the Reynold's number which can be calculated using dynamic viscosity (μ) of the fluid as,

$$N_{Re} = \frac{\rho v D_h}{\mu} \tag{13}$$

However, Equation 12 is an implicit function of f_d and it has to be solved iteratively (increases computation time). Instead, approximation to Equation 12 by using Haaland equation can be used to calculate f_d as,

$$\frac{1}{\sqrt{f_d}} = -1.8\log_{10}\left[\frac{6.9}{N_{Re}} + \left(\frac{\epsilon/D_h}{3.7}\right)^{1.11}\right] \tag{14}$$

In the annulus, the drill fluid will be mixed with the fluid produced from the reservoir with given water cut. The impact of water cut on the viscosity of the fluid can be expressed using Brinkman formula [8] as,

$$\mu_r = (1 - W_c)^{-2.5} \tag{15}$$

Here, μ_r is the relative viscosity. If μ_d is the dynamic viscosity of the drill mud in the drill string, the dynamic viscosity of the mixture of fluids in the annulus can be calculated as,

$$\mu_a = \mu_r u_d \tag{16}$$

Again remember that if $q_{res} = 0$, then $W_c = 0$. In such condition, $u_a = \mu_d$.

Note: We could use Haaland equation as described above to calculate the friction factor, but for lower values of $q_{pump} \approx 0$, the approximation formula of Equation 14 may pose numerical difficulties and instability. Thus for this group project, it is recommended that the students use $f_d = 0.02$ for the pipelines instead of using Equation 14.

3. Pipe Connection during drilling

During drilling, there could be many disturbances that can cause fluctuations in the bottom hole pressure of the annulus [1]. One source of such a disturbance that is considered for the group project is caused by the pipe connection procedure. The drill pipes come in stands of about 27 m. As the drilling proceeds, the well will become longer. New pipe segments should be regularly added to form the drill string (the drill string consists of several pipe segments which are joined together). During the pipe connection procedure, the rotation of the drill bit should be stopped. At the same time, pumping of the drill fluid into the drill string is stopped (ramped down to zero). A new pipe segment is then added and the mud pump is re-started (ramped up to nominal value). The drill bits are rotated again and the drilling continues. The connection procedure is repeated when another new pipe segment is added. The pipe connection procedure severely fluctuates the bottom hole pressure of the annulus.

Let us look at an example shown in Figure 3. Two pipe connections are considered, first connection at *t =10 minutes* and the second connection at *t = 45 minutes.* As can be seen, at *t =10 minutes* the flow rate through the mud pump is ramped down from 1500 l/min to 0 l/min. After the pipe connection procedure is completed at *t =25 minutes*, the flow rate through the mud pump is ramped up from 0 l/min to 1500 l/min.

Clearly from Figure 3, it can be seen that during the pipe connection procedure, the bottom hole pressure of the annulus can fall below the collapse pressure (the lower limit of the drilling window). This should be avoided because the walls of the well bore may collapse, the drill string may get stuck and become unrecoverable. A new well should be drilled again (adding expenses to drilling procedure).

Assume that the drilling is being performed near/at the reservoir zone. During the pipe connection procedure, the flow of fluid from the reservoir into the annulus (reservoir influx) is increased to about 600 l/min (see Figure 3). If this is not controlled, blow out of the well might occur leading to environmental damage and possible loss of lives.

In this project work, students should develop a nonlinear model predictive controller to keep the bottom hole pressure of the annulus within the drilling window. The back pressure pump is not always available in all of the drilling rigs (especially in the older drilling rigs, it is not present), but for this group project, we assume that it is present in the drilling rig. However, choke valve is always present in all the drilling rigs. For the pipe connection procedure, the drilling is stopped and there will be no penetration. You can use $v_{pen} = 0$.

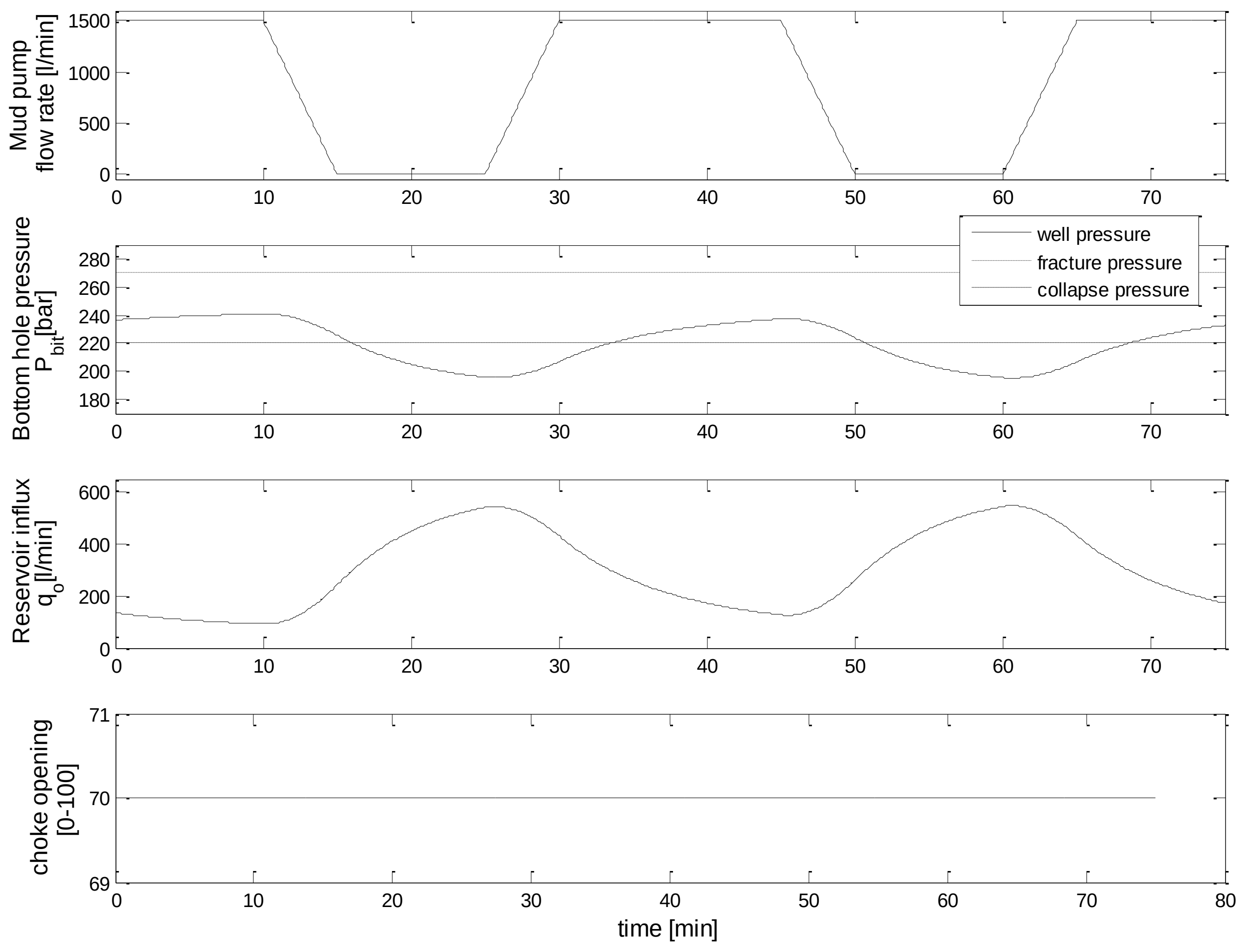

Figure 3: Open loop simulation of the drilling model without control actions

4. Tasks

The following tasks should be performed for this lab work. Use the parameters that are listed in Table 1. Assume that the states and the output variables of the system are measurable or known perfectly (i.e. no state estimation is performed here).

i) Write down a model summary by collecting all the equations of the drilling operation. If you find anything erroneous or missing, please correct/assume it.

ii) Using MATLAB/Simulink, perform the open loop simulation (without any controllers or feedback) for the pipe connection procedure using the model that you listed in step (i). Keep in mind the following sub-points:

a. Do not perform severe (or large) step changes of the mud pump flow rate i.e. do not directly shut down the pump from 1500 l/min to 0 l/min in one time step. This is rarely done in practice. Instead, ramp down (while shutting down) or ramp up (while starting up) the flow rate through the mud pump. Assume that it takes at least 5 minutes to change the flow rate of the mud pump from 1500 l/min to 0 l/min and vice versa.

b. For open loop simulation, use $u_c = 70$ and $q_{back} = 0$.

c. Plot all the variables of interest ($q_{pump}, P_{bit}, q_o, u_c$ $etc.$) and any other variables that you find it interesting to study. It should look similar to Figure 3.
d. Discuss the simulation results.

Table1: Values for the parameters and variables

Parameters /Variables	Value	Unit	Description
ρ_l	1150	kg/m^3	Drill mud density
ρ_w	1000	kg/m^3	Water density
WC	0.1		Water cut of the reservoir fluid
A_d	0.0067	m^2	Cross sectional area of drill string
A_a	0.278	m^2	Cross sectional area of annulus
D_d	0.0925	m	Hydraulic diameter of drill string
D_a	0.211	m	Hydraulic diameter of annulus
L	1600	m	Vertical depth of the well
PI	1.6667×10^{-9}	m^5/Ns	Productivity Index value
P_{res}	250×10^5	N/m^2	Reservoir pressure
P_{frac}	270×10^5	N/m^2	Fracture pressure
P_{coll}	220×10^5	N/m^2	Collapse pressure
P_{bit}^{ref}	Your choice	N/m^2	Reference pressure
P_0	4×10^5	N/m^2	Pressure downstream the choke valve
ϵ/D_d	10^{-5}		Relative roughness of pipe in drill string
ϵ/D_a	10^{-4}		Relative roughness of pipe in the annulus
μ_d	0.015	kg/ms	Dynamic viscosity of the drill fluid
β_d	3×10^8	N/m^2	Bulk modulus in the drill string
β_a	2.4×10^8	N/m^2	Bulk modulus in the annulus
q_{pump}^{nom}	1500	l/min	Nominal flow rate of the drill fluid
u_c^{nom}	70	%	Nominal choke valve opening
g	9.81	m/s^2	Acceleration due to gravity
f_d	0.02		Friction factor

Good luck!!

References:

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