Exercise 2

LQ optimal control of an inverted pendulum in Simulink using *qpOASES solver*

Aim:

To be familiar with the formulation of LQ optimal control for a dynamic system. To be familiar with the use of qpOASES solver in Simulink for solving LQ optimal control problems.

1. Preliminary requirements

You should have understood the contents of lecture 3. If not, make sure that you have read and looked into **all** the videos connected to lecture3. You should also have looked into how we can formulate the LQ optimal control problem to the standard QP problem (to the structure that solver *qpOASES* supports) using Kronecker products. Please also see lecture notes chapter 3.5 for the inverted pendulum example and code snippets.

To solve this exercise, you should have installed both the *qpOASES* solver and the C++ compiler. Please look at the videos for lecture 2 in the homepage (https://home.usn.no/roshans/mpc) if you have not done so.

2. Design and implement an LQ optimal controller for controlling the inverted pendulum.

The mechanistic model of the inverted pendulum is by the following set of nonlinear ODEs.

$$\frac{d\alpha}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{m_1+m_2}{m_1^2 l^2 cos^2\alpha - m_1^2 l^2 - m_1 m_2 l^2}(k_T l^2 |\omega|\omega - m_1 g l\, sin\alpha) + \frac{cos\alpha}{m_1 l cos^2\alpha - m_1 l - m_2 l}(F + \omega^2 m_1 l\, sin\alpha)$$

$$\frac{dx_2}{dt} = v_2$$

$$\frac{dv_2}{dt} = \frac{1}{m_1 l\, cos\alpha}\left(m_1 g l\, sin\alpha - k_T l^2 |\omega|\omega - m_1 l^2 \frac{d\omega}{dt}\right)$$

The parameters of the system are: $m_1 = 1\ \text{kg}, m_2 = 2\ \text{kg}, l = 1\ \text{m}, k_T = 0.1\ \text{kg/rad}^2$.

Since, we will be designing an LQ optimal controller, we need a linear model of the inverted pendulum system. The nonlinear ODEs can be linearized around the equilibrium point ($\alpha_s = 0, w_s = 0, x_{2s} = 0$ and $v_{2s} = 0$). We will then obtain a continuous time linear model of the system as,

$$\frac{dx}{dt} = A_c x(t) + B_c u(t) \tag{3.28}$$

(3.29)

$$y = C_c x(t)$$

where,

$$x = \begin{bmatrix} \alpha \\ \omega \\ x_2 \\ v_2 \end{bmatrix}, A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 14.715 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -4.905 & 0 & 0 & 0 \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ -0.5 \\ 0 \\ 0.5 \end{bmatrix}, C_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Task:

We will control the angle of the inverted pendulum so that it remains inverted (i.e. it stands upright with $= 0$). At the same time we will also control the position of the cart. In order to control α and x_2, the force acting on the cart(F) can be adjusted/manipulated. Thus the control input of the system is F.

Clearly, the setpoint for the inverted pendulum angle is 0 throughout the prediction horizon. The set point for the cart position x_2 can be chosen as shown in Figure 1.

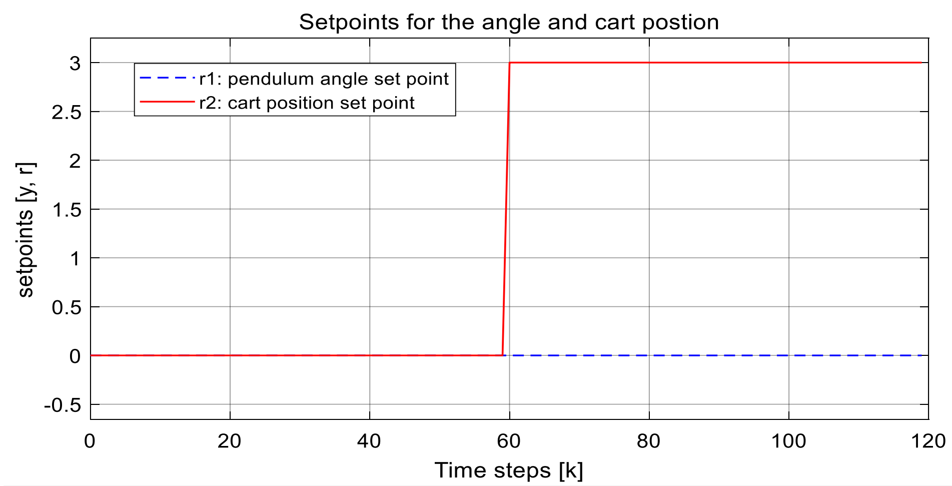

Figure 1: Setpoints for the angle of pendulum and position of the cart

From Figure 1, you can also see that the prediction horizon for this task is $N = 120$. You can take $dt = 0.1$ as the sampling time.

Following the example described in Chapter 3.5 in lecture notes, **design and implement an LQ optimal controller for controlling the pendulum angle and the cart position to their respective setpoints of Figure 1.**

Show your objective function and constraints clearly. Use Simulink to implement your controller. Use *qpOASES* solver for obtaining the optimal solution.

Individual submission: Submit a lab journal with screenshots of your implementation and the solution. You should submit your lab journal in Canvas. Make sure to show all the details of your implementation (i.e. show what is inside your subsystem blocks).

Good luck!

Things to remember:

a) Make sure that you compile the *qpOASES* solver so that it matches your problem size.
b) You should ofcourse use discrete time model. So you should discretize your continuous time model.
c) Formulate the QP problem using kronecker products. Form all the necessary matrices like H, c, A_e, b_e, z_L, z_U etc.