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Labwork 1

Simulation of 2 DOF freedom helicopter model in Simulink

Aim:

a) To be familiar with the use of Simulink for making a simulator to simulate a dynamic system whose mathematical model is provided.
b) To linearize the nonlinear model, simulate the linear model and to compare the results with between the linear and nonlinear model. This simulator can be later on used for the project part of the course (so keep your files safe).

1. Preliminary requirements

You should read the description of the 2 DOF helicopter model and be familiar with the mathematical model of the system. Please read the document
https://web01.usn.no/~roshans/mpc/downloads/description_helicopter.pdf

You should also have already seen the following two videos (examples of simulating an inverted pendulum):

https://web01.usn.no/~roshans/mpc/videos/lecture1/openloop-simulation-pendulum.mp4

https://web01.usn.no/~roshans/mpc/videos/lecture1/realtime-simulation-pendulum.mp4

2. Mathematical model of the helicopter unit

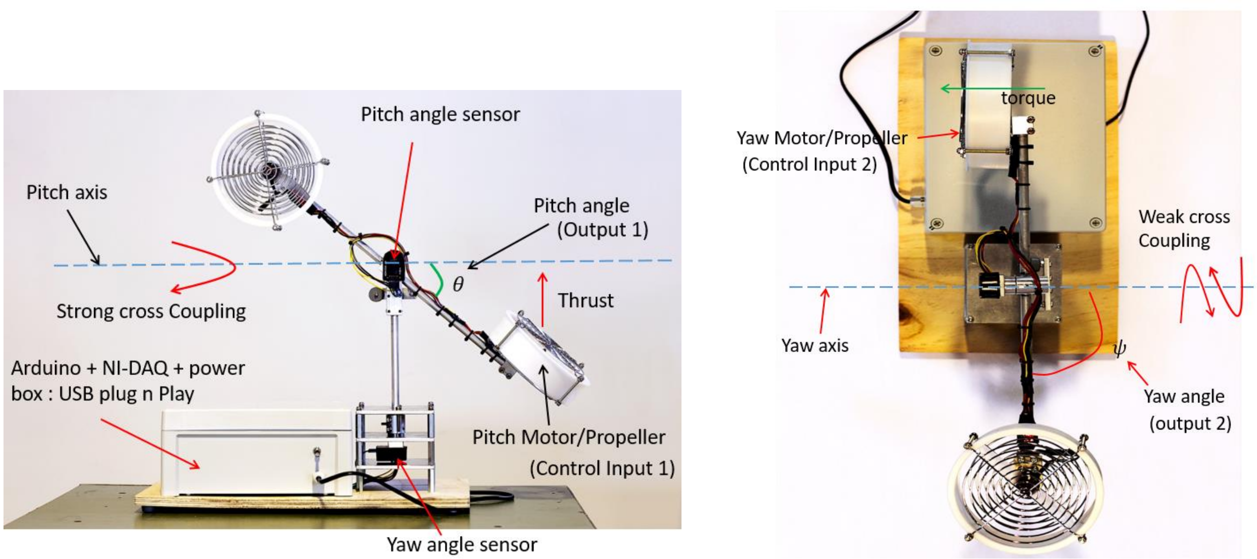

Figure 1: 2-DOF helicopter unit at USN (side view and top view)

Let us define the following:

<u>Control Inputs:</u>

V_{mp} := voltage applied to the picth motor, V_{my} := voltage applied to the yaw motor

Measured Outputs:

θ := pitch angle, ψ := yaw angle

The system can be described with four states:

θ := pitch angle ψ := yaw angle

ω_θ := pitch angular velocity ω_ψ := yaw angular velocity

The block diagram of the system showing the inputs, outputs and the states is shown in Figure 2.

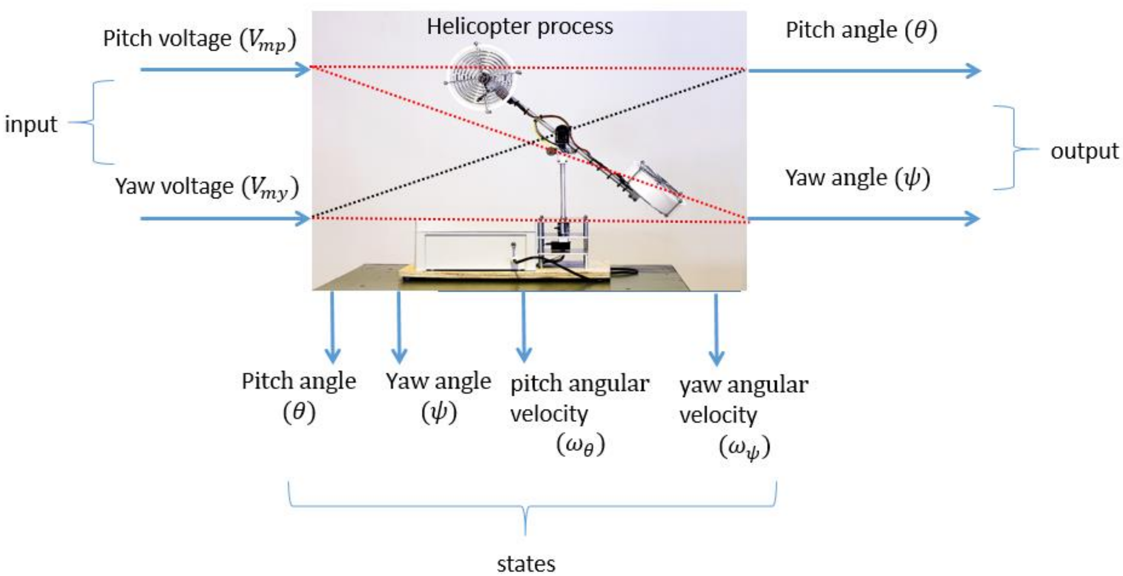

Figure 2: Block diagram of the system showing inputs, outputs and states

The model is described by a set of four ordinary differential equations (ODEs).

$$\frac{d\theta}{dt} = \omega_\theta \tag{1}$$

$$\frac{d\psi}{dt} = \omega_\psi \tag{2}$$

$$\frac{d\omega_\theta}{dt} = \frac{K_{pp}V_{mp}}{J_{eq,p} + m_{heli}\, l_{cm}^2} - \frac{K_{py}V_{my}}{J_{eq,p} + m_{heli}\, l_{cm}^2} - \frac{B_p\omega_\theta + m_{heli}\,\omega_\psi^2\,\sin(\theta)\, l_{cm}^2 \cos(\theta) + m_{heli}\, g\cos(\theta)\, l_{cm}}{J_{eq,p} + m_{heli}\, l_{cm}^2} \tag{3}$$

$$\frac{d\omega_\psi}{dt} = \frac{K_{yp}V_{mp}}{J_{eq,y} + m_{heli}\cos^2(\theta)\, l_{cm}^2} - \frac{K_{yy}V_{my}}{J_{eq,y} + m_{heli}\cos^2(\theta)\, l_{cm}^2} + \frac{2m_{heli}\,\omega_\psi \sin(\theta)\, l_{cm}^2\cos(\theta)\, w_\theta}{J_{eq,y} + m_{heli}\cos^2(\theta)\, l_{cm}^2} - \frac{B_y\omega_\psi}{J_{eq,y} + m_{heli}\cos^2(\theta)\, l_{cm}^2} \tag{4}$$

Table 1: Parameters of the system: part 1

Parameter	Description	Value
l_{cm}	Distance between the pivot point and the center of mass of helicopter	0.015 [m]
m_{heli}	Total moving mass of the helicopter	0.479 [kg]
$J_{eq,p}$	Moment of inertia about the pitch axis	0.0172 [kg-m^2]
$J_{eq,y}$	Moment of inertia about the yaw axis	0.0210 [kg-m^2]
g	Acceleration due to gravity on planet earth	9.81 [m-s^{-2}]

Table 2. Parameters of the system: part2

Parameter	Description	Value
K_{pp}	Torque constant on pitch axis from pitch motor/propeller	0.0556 [Nm/V]
K_{yy}	Torque constant on yaw axis from yaw motor/propeller	0.21084 [Nm/V]
K_{py}	Torque constant on pitch axis from yaw motor/propeller	0.005 [Nm/V]
K_{yp}	Torque constant on yaw axis from pitch motor/propeller	0.15 [Nm/V]
B_p	Damping friction factor about pitch axis	0.01 [N/V]
B_y	Damping friction factor about yaw axis	0.08 [N/V]

3. Tasks

The following two tasks should be completed.

Task 1:

Openloop simulation of the helicopter model in Simulink with real time pacer

By following the example videos, please make a simulator for simulating the mathematical model (equations 1 -4 and the parameters in Tables 1 & 2) of the helicopter unit in Simulink. Use real time pacer for real time simulation of the model.

Some useful information:

a) The units of the pitch and the yaw angles are in radians and all calculations should be performed in radians. However, it is easier to view the plots in degrees. So, when plotting θ and ψ, please use degrees i.e. convert radians to degrees only for plotting.
b) The range of the input voltage is from 0 [V] to 5 [V]. But for normal operation (when you play around the input voltages manually), the input voltage for the pitch motor can be between 1.1 [V] to 1.4 [volts]. The normal operating input voltage for the yaw motor can be between 0.4 [V] to 1.3 [V].
c) The pitch angle cannot be less than -45 degrees and cannot be more than +45 degrees i.e. $-45^0 \leq \theta \leq 45^0$. Please use saturation on the integrators by double clicking the integrator blocks and limiting the output. But of course, actual unit is in radians.
d) The yaw angle cannot be less than 0 degrees and cannot be more than 180 degrees i.e. $0^0 \leq \psi \leq 180^0$. Please use saturation on the integrators appropriately. Please use saturation on the integrators by double clicking the integrator blocks and limiting the output. But of course, actual unit is in radians.

e) Initial values of the states can be $[-\frac{45\pi}{180}, 0, 0, 0]$ for $\theta, \psi, \omega_\theta$ and ω_ψ respectively. Please initialize your integrators appropriately.

Individual submission: Submit a lab journal with screenshots of your implementation and various plots. You should submit your lab journal in Canvas. Make sure to show all the details of your implementation (i.e. show what is inside your subsystem blocks).

For inspiration, have a look at this simulator of the helicopter unit and try to make something similar:

https://web01.usn.no/~roshans/mpc/downloads/helicopter_openloop_simulator.zip

Task 2:

Linearization of the nonlinear model of the helicopter unit by Taylor series and simulation of linear model

The linear model of the helicopter can be obtained by linearizing the nonlinear model of Equations 1- 4. To linearize a model, we should at first select the point of linearization (also known as the operating points). For the helicopter unit, you can choose the operating points for the states to be as follows:

$$\theta_{op} = -\frac{10\pi}{180}, \qquad \psi_{op} = \frac{\pi}{2}, \qquad \omega_{\theta,op} = 0 \quad \text{and} \qquad \omega_{\psi,op} = 0$$

Here the subscript *'op'* means operating point. The corresponding values of the operating points for the control inputs can be calculated as,

$$V_{mp,op} = \frac{K_{yy} m_{heli}\, g \cos(\theta_{op})\, l_{cm}}{(K_{yy}K_{pp} - K_{yp}K_{py})}$$

$$V_{my,op} = \frac{K_{yp} V_{mp,op}}{K_{yy}}$$

The linear model of the helicopter is then given by the following state space model in the deviation form:

$$\delta\dot{x} = A_c \delta x + B_c \delta u \qquad (5)$$

$$\delta y = C_c \delta x \qquad (6)$$

Here,

$$\delta x = \begin{bmatrix} \theta - \theta_{op} \\ \psi - \psi_{op} \\ \omega_\theta - \omega_{\theta,op} \\ \omega_\psi - \omega_{\psi,op} \end{bmatrix} \quad \text{i.e. the deviation of the states from its operating points}$$

$$\delta u = \begin{bmatrix} V_{mp} - V_{mp,op} \\ V_{my} - V_{my,op} \end{bmatrix} \quad \text{i.e. the deviation of the control inputs from its operating point voltage}$$

For simulating the linear ODEs in Simulink we need the initial values of the states in the deviation form. You can take the initial values of the deviation states to be zero for each of the states.

An example of the linearization of an inverted pendulum nonlinear model is given in this video

https://web01.usn.no/~roshans/mpc/videos/lecture1/linearization-pendulum.mp4

Note: For the given operating point $x_{op} = [-\frac{10\pi}{180}, \frac{\pi}{2}, 0, 0]$ for the helicopter model, you **cannot** make the assumption that $\sin(\alpha) \approx \alpha$ and $\cos(\alpha) \approx 1$ as in this case $\alpha \neq 0$. So please work out your way up from the scratch using basic trigonometry and calculus.

Now, your task is to linearize the helicopter model of Equations 1 – 4 at the given operating point (or linearization point) and generate the matrices A_c, B_c, and C_c.

Simulate the linear model together with the nonlinear model using Simulink. Comment on the response that you get from the linearized model and the nonlinear model when the same control input signals are applied to them.

Also try to go far away from the operating point and observe the dynamics from the linearized model and from the nonlinear model.

Individual submission: Please take a snap shot of the A_c, B_c, and C_c matrices and submit it in Canvas. Properly describe the comparison of the solution from the nonlinear model with the linear model.

Some notes:

Very special care should be taken while simulating the linear state space model.

a) If you look at the linear model of the system ($\frac{d\delta x}{dt} = \delta\dot{x} = A_c\delta x + B_c\delta u$), you can see that it contains δu, but not u. So the linear model should be excited with the deviation of the control inputs i.e. you should apply ($V_{mp} - V_{mp,op}$) and ($V_{my} - V_{my,op}$) as the two control inputs to your linear model. The nonlinear model should be however excited with V_{mp} and V_{my}.
b) Although, the linear model (equation 5) has only single equation, you should remember that it is in compact form and it already contains all the four states of the system in the deviation form. You can use MATLAB function block in Simulink to write down the right hand side of linear equation 5. From the MATLAB function block you can return back (or pass out) the left hand side of equation 5 (e.g. as *d_delta_x*). Outside the MATLAB function block, you should then split up *d_delta_x* signal into four parts using demuxer. Then you should use integrators (four separate integrators, one for each) to integrate all the deviation states of the system. Out from the integrators you will get ($\delta\theta, \delta\psi, \delta\omega_\theta$ and $\delta\omega_\psi$), but not $\theta, \psi, \omega_\theta$ and ω_ψ. You should take back ($\delta\theta, \delta\psi, \delta\omega_\theta$ and $\delta\omega_\psi$) to feed it into your MATLAB function block for linear model.
Thus, you get $\delta\theta, \delta\psi, \delta\omega_\theta$ and $\delta\omega_\psi$ as the solution from the linear model after the integration is performed. However, if you want to calculate actual values of the states ($\theta, \psi, \omega_\theta$ and ω_ψ) from the linear model, then you should add the operating point values of the states to $\delta\theta, \delta\psi, \delta\omega_\theta$ and $\delta\omega_\psi$ i.e.
$$\theta = \delta\theta + \theta_{op}, \quad \psi = \delta\psi + \psi_{op}, \quad \omega_\theta = \delta\omega_\theta + \omega_{\theta,op}, \ \omega_\psi = \delta\omega_\psi + \omega_{\psi,op}$$
c) After properly taking care of points (a) and (b), finally you can compare the actual levels from the nonlinear model and the linear model at the same time.

Good luck!