

## Group Project Description

### Design and implementation of MPC for controlling two degrees of freedom (2-DOF) helicopter unit

#### Choice 1

#### 1. Introduction/Information

The group project for the course IIA4117 (Model Predictive Control) is an obligatory task that all the students enrolled in this course should fulfill. It counts 40% of the course grading. This is a group project and each student should be a part of a group. The students have the responsibility to form the groups.

A technical report should be delivered by each group. The deadline for submitting the report and the date for the demonstration is given at the end of the course homepage (<https://web01.usn.no/~roshans/mpc>) (scroll to the bottom). Each group will receive a score (from 0 to 40) for the project work. Since there will be no presentation and demonstration, the grade for the project work is solely based on the quality of the submitted report. So make sure that you submit a good report.

#### 2. Process description

The 2-DOF helicopter unit developed at USN is a dynamic system with multiple inputs and multiple outputs (MIMO). Figure 1 shows the schematic of such a unit with the side view and the top view. It consists of two propellers (pitch and yaw) driven by motors.

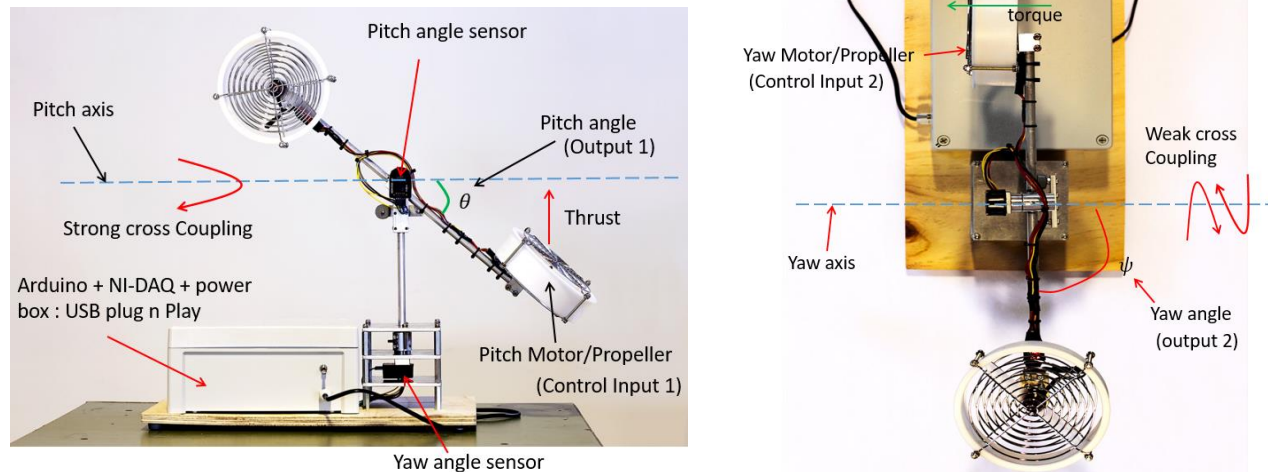


Figure 1: 2-DOF helicopter unit at USN (side view and top view)

The unit has two inputs: (a) voltage to the front or pitch motor/propeller system, and (b) voltage to the back or yaw motor/propeller system. When voltage is applied to the pitch motor, the pitch propeller rotates and it generates thrust, and the helicopter lifts up. Thus voltage to the pitch motor/propeller control the elevation (or pitch) of the helicopter nose about the pitch axis. When voltage is applied to the yaw motor, the yaw

propeller rotates and it generates torque in anti-clockwise direction, and the helicopter rotates about the yaw axis. The angle between the pitch axis and the helicopter body axis is called the pitch angle. The angle between the yaw axis and the helicopter body axis is called the yaw angle. The pitch and the yaw angles are measured by using the angle sensors as shown in Figure 1. Thus, these are the two outputs of the system which are measurable.

Let us define the following:

Inputs:

$V_{mp}$  := voltage applied to the pitch motor

$V_{my}$  := voltage applied to the yaw motor

Outputs:

$\theta$  := pitch angle

$\psi$  := yaw angle

The process is a cross-coupled MIMO system. When sufficient voltage is applied to the front motor, the helicopter not only pitches up but it also starts to rotate at the same time i.e. the input  $V_{mp}$  affects both outputs  $\theta$  and  $\psi$ . Similarly, when sufficient voltage is applied to the back motor, the helicopter rotates in the anti-clockwise direction and at the same time, it also changes its pitch a little i.e. the input  $V_{my}$  affects both outputs  $\theta$  and  $\psi$ . The effect of  $V_{mp}$  on  $\psi$  is very strong denoted by strong cross-coupling in Figure 1, while the effect of  $V_{my}$  on  $\theta$  is weak denoted by weak cross-coupling.

The system can be described with four states:

$\theta$  := pitch angle

$\psi$  := yaw angle

$\omega_\theta$  := pitch angular velocity

$\omega_\psi$  := yaw angular velocity

The block diagram of the system showing the inputs, outputs and the states is shown in Figure 2. For detailed description of the helicopter see the video at

<https://web01.usn.no/~roshans/mpc/videos/labworks/helicopter-description.mp4>.

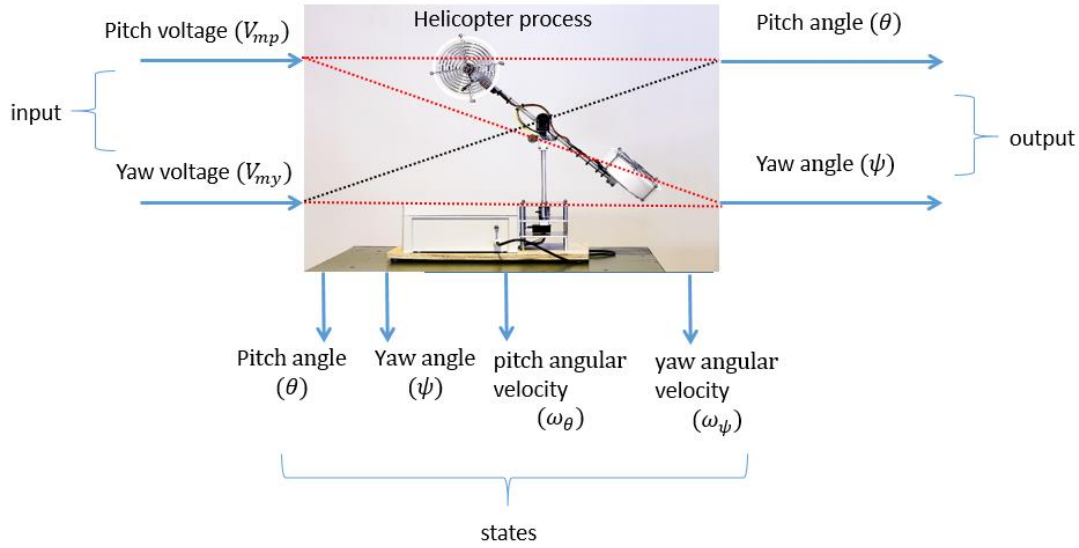


Figure 2: Block diagram of the system showing inputs, outputs and states

### 3. Mathematical model of the process

The dynamics of the 2-DOF helicopter unit is modelled using Newton's laws of motion and Euler-Lagrange equations. The model is described by a set of four ordinary differential equations (ODEs).

$$\frac{d\theta}{dt} = \omega_{\theta} \quad (1)$$

$$\frac{d\psi}{dt} = \omega_{\psi} \quad (2)$$

$$\frac{d\omega_{\theta}}{dt} = \frac{K_{pp}V_{mp}}{J_{eq,p} + m_{heli} l_{cm}^2} - \frac{K_{py}V_{my}}{J_{eq,p} + m_{heli} l_{cm}^2} - \frac{B_p\omega_{\theta} + m_{heli} \omega_{\psi}^2 \sin(\theta) l_{cm}^2 \cos(\theta) + m_{heli} g \cos(\theta) l_{cm}}{J_{eq,p} + m_{heli} l_{cm}^2} \quad (3)$$

$$\frac{d\omega_{\psi}}{dt} = \frac{K_{yp}V_{mp}}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} - \frac{K_{yy}V_{my}}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} + \frac{2m_{heli} \omega_{\psi} \sin(\theta) l_{cm}^2 \cos(\theta) \omega_{\theta}}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} - \frac{B_y\omega_{\psi}}{J_{eq,y} + m_{heli} \cos^2(\theta) l_{cm}^2} \quad (4)$$

There are altogether 11 parameters of the system.  $l_{cm}$  is a parameter which denotes the distance of the pivot point from the center of mass as shown in Figure 3. The helicopter rotates freely (up/down and clock/anti clock wise) about the pivot point. Center of mass is the point at which the gravitational force act on the system.

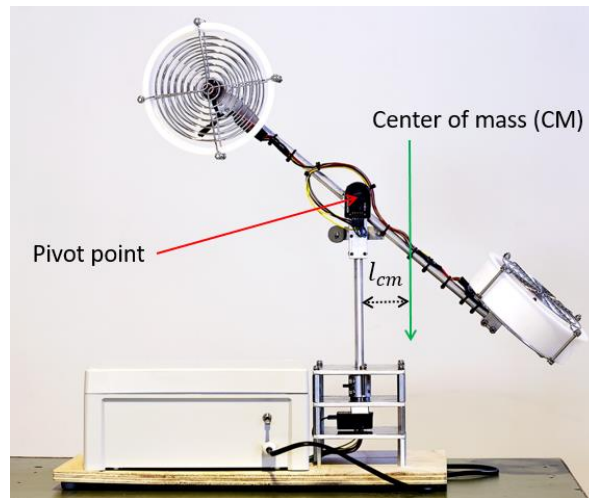


Figure 3: Illustration of center of mass and pivot point

All the helicopter units at USN are fastened and screwed in a way that  $l_{cm} = 1.5 [cm]$ . For a fixed value of  $l_{cm}$  and  $m_{heli}$ , the moment of inertia can be calculated and have been listen in Table 1.

Table 1: Parameters of the system: part 1

Parameter	Description	Value
$l_{cm}$	Distance between the pivot point and the center of mass of helicopter	0.015 [m]
$m_{heli}$	Total moving mass of the helicopter	0.479 [kg]
$J_{eq,p}$	Moment of inertia about the pitch axis	0.0172 [kg-m <sup>2</sup> ]
$J_{eq,y}$	Moment of inertia about the yaw axis	0.0210 [kg-m <sup>2</sup> ]
$g$	Acceleration due to gravity on planet earth	9.81 [m-s <sup>-2</sup> ]

The remaining 6 parameters of the system namely,  $K_{pp}$ ,  $K_{yy}$ ,  $K_{py}$ ,  $K_{yp}$ ,  $B_p$  and  $B_y$  are slightly different for each unit of the helicopter at USN. For one of the unit, Table 2 lists the values of these parameters. You can use the parameters listed in Table 2 for simulations.

Table 2. Parameters of the system: part2

Parameter	Description	Value
$K_{pp}$	Torque constant on pitch axis from pitch motor/propeller	0.0556 [Nm/V]
$K_{yy}$	Torque constant on yaw axis from yaw motor/propeller	0.21084 [Nm/V]
$K_{py}$	Torque constant on pitch axis from yaw motor/propeller	0.005 [Nm/V]
$K_{yp}$	Torque constant on yaw axis from pitch motor/propeller	0.15 [Nm/V]
$B_p$	Damping friction factor about pitch axis	0.01 [N/V]
$B_y$	Damping friction factor about yaw axis	0.08 [N/V]

#### 4. TASKS

The task will be divided into two major parts:

- (i) Development of simulator
- (ii) Implementation to a real helicopter unit (optional for students, i.e. it is not compulsory, but I strongly recommend the campus students to try to fulfill this task).

#### 4.1 Simulator development

For developing simulator(s), you can work on your computer(laptops) with Simulink/MATLAB. You do not need access to the real process. The following tasks should be completed:

- (i) Describe the 2 DOF helicopter process. E.g. How does the process operate? What are the inputs, outputs and states of the system? You can re-use the figures/pictures that is available to you or you can take/make your own figures.
- (ii) List all the equations involved with the mathematical model of the process. Describe a little about the mathematical model. E.g. What kind, type? What is its order? linear or nonlinear etc.
- (iii) Using Simulink, perform the openloop simulation of the nonlinear model of the process. Manually excite your system with different values of the control inputs and see the response of the system. Plot the necessary variables of interests (e.g. angles, velocities, control inputs etc.). Discuss your simulation results.
- (iv) Choose a suitable operating point and linearize the model of the process about this operating point. Show all the necessary calculations involved during the linearization of the mathematical model.

To linearize your model, first choose a suitable operating point for the four states (for e.g.  $(-\frac{10\pi}{180}, \frac{\pi}{2}, 0, 0)$  for pitch angle, yaw angle, pitch velocity and yaw velocity respectively). To find the corresponding operating point for the control inputs, solve the nonlinear process model at the steady state for  $V_{mp}$  and  $V_{my}$  using the operating point for the states, i.e. set the left hand side of equations (1) to (4) to zero solve these equations (by using the operating point of the states) for calculating operating point for control input voltages.

- (v) Using Simulink, simulate the linear model alongside the nonlinear model. Compare the behavior of the process from these two models. Comment and discuss about your simulation results.
- (vi) Assume that you can measure only the two angles (the pitch angle ( $\theta$ ) and the yaw angle ( $\psi$ )). Design a suitable state estimator in Simulink for estimating the remaining two states (the pitch velocity ( $\omega_\theta$ ) and the yaw velocity ( $\omega_\psi$ )). Here you are not limited to using steady state Kalman filter but you can use any other state estimator that you know if you want. Change the control inputs manually and observe the behavior of your state estimator. Discuss and comment in detail about your simulation results.
- (vii) Design an output feedback linear MPC for controlling the pitch angle ( $\theta$ ) and the yaw angle ( $\psi$ ) to their respective setpoints. At this point, do not worry about integral action, however, please do consider the following:
  - a. Your controller should not supply more than 3[V] as the control inputs. They cannot be less than 0 [V] either. Using signal limiter block in Simulink is NOT ENOUGH. You should consider this while formulating your optimal control problem.
  - b. You should show the MPC problem formulation very clearly. Show the objective function, constraints, bounds etc. Show your choice of the unknown vectors to optimize.

- c. You should show every detailed step of how you calculated/formulated all the necessary matrices and vectors. How did you handle the bounds (if any?)
  - d. How did you implement the receding horizon strategy in Simulink? Show clearly.
  - e. For the simulator, consider the nonlinear model of the helicopter as your process or target system. Please DO NOT use the linear model as your process. You can of course make use of things that you already did in task (iii) for the nonlinear model of the process.
  - f. Provide step changes to the setpoint values and observe the response of your controller. Change your setpoint as ramp (for e.g. using slider or knob in Simulink) and observe the response of your controller.
  - g. Plot all the necessary variables (outputs, estimated states, inputs, references etc.)
  - h. Discuss in details all your simulation results.
  - i. Among many other things, did you observe any steady-state offsets with your linear MPC?
- (viii) Design an output feedback linear MPC with integral action. Consider the points (vii) a, e, f and g while designing the controller. How can you ensure integral action with your linear MPC? If you know many ways to achieve integral action, just describe the one that you have used in your project. Discuss in detail your simulation results.

#### **4.2 Implementation on a real helicopter unit (optional for the students, no need to do if you do not want to do. But I strongly recommend the campus students to fulfill this task if they can.)**

After you think that your simulators are working as expected, you can then implement your linear MPC on a real helicopter unit. The campus students have a golden opportunity to use the units at USN right from the beginning of the project work. For the online students, contact the course instructor if you want to do implementation/testing of your algorithms with a real unit. Please fulfill the following:

- (i) Implement the state estimator on the real helicopter unit for estimating pitch velocity ( $\omega_\theta$ ) and the yaw velocity ( $\omega_\psi$ ). Plot all the necessary variables (e.g. estimated states, outputs, inputs etc.). Change the control inputs manually and observe the behavior of your state estimator. Discuss and comment in detail about your results.
- (ii) Implement a linear MPC with integral action to the real helicopter unit. Provide step changes to the setpoint values and observe the response of your controller. Change your setpoint as ramp (for e.g. using slider or knob in Simulink) and observe the response of your controller. Plot all the necessary variables (outputs, estimated states, inputs, references etc.). Discuss in details all your results.

Good luck!